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RESEARCH ARTICLE

Generalised roughness in $(\in,\in\vee q)$-fuzzy substructures of LA-semigroups

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Abstract: LA-semigroups are non-associative sturctures of great importance. Study of generalised roughness for fuzzy algebraic substrucures of LA-semigroups has been initiated. Many different kinds of set valued maps are needed to preserve an algebraic substrucure, while considering its lower and upper approximations. In the present paper generalised lower and upper approximations in $(\in,\in\vee q)$-fuzzy ideals of LA-semigroups have been investigated. An $(\in,\in\vee q)$-fuzzy subset of an LA-subsemigroup has two parts, *viz*. lower and upper parts. Many properties of lower and upper approximations have been given for these. In conclusion, lower and upper approximations for $(\in,\in\vee q)$-fuzzy interior ideals and $(\in,\in\vee q)$-fuzzy bi-ideals have been discussed in LA-semigroups.

Keywords: Generalised roughness, LA-semigroups, rough set.

INTRODUCTION

The algebraic structure of a left almost semigroup is abbreviated as an LA-semigroup introduced by Naseerudin and Kazim (1972). Later Mushtaq and others investigated the structure of LA-semigroups and added some other important results to literature (Mushtaq, 1983; Mushtaq & Iqbal, 1991; Mushtaq & Khan, 2009).

Zadeh introduced the notion of fuzzy subset (Zadeh, 1965). Rosenfeld inspired the fuzzification of algebraic structures and introduced the concept of fuzzy subgroup of a group (Rosenfeld, 1971). Kuroki (1979) initiated the study of fuzzy semigroups.

The idea of quasi coincidence of a fuzzy point with a fuzzy set, which is mentioned in Pu and Liu (1980), played a vital role to generate some different types of fuzzy subgroups called (*α, β*)-fuzzy subgroups, introduced by Bakat and Das (1992; 1996a; 1996b). Fuzzy point played a vital role in the study of (*α, β*)-fuzzy subgroups initiated by Bhakat and Das (1996) using the combined notions of 'belongingness' and 'quasi-coincidence' of a fuzzy point with a fuzzy set. Khan *et al*. (2010a; 2010b) applied this concept in AG-groupoids. Shabir *et al*. (2010) have applied this concept in semigroups. Rehman and Shabir (2012) initiated the study of (*α, β*)-fuzzy substructure in ternary semigroups. Khan *et al.* (2010a; 2010b) introduced $(\in,\in\vee q)$-fuzzy ideals in AG-groupoids.

Pawlak (1982) was the first to discuss rough set with the help of equivalence relation among the elements of a set, which is a key point to discuss the uncertainty. There are at least two methods for the development of rough set theory: the constructive and axiomatic approaches. In constructive methods, lower and upper approximations are constructed from the primitive notions, such as equivalence relations on a universe and a neighbourhood system. In rough sets, equivalence classes play an important role in the construction of both lower and upper approximations. However, sometimes in algebraic structures, finding equivalence relations is too difficult just like in LA-semigroups. Many authors have worked on this to initiate rough set without equivalence relations. Couso and Dubois (2001) initiated generalised rough set

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or T-rough set with the help of a set valued mapping. It is a more generalised rough set form than the Pawlak rough set. The notion of roughness in fuzzy set was introduced by Dubois and Prade (1990). Some researchers applied this concept in their work (Banerjee & Pal, 1996; Chakrabarty *et al*., 2000).

Many researchers have taken interest to apply the concept of roughness in different algebraic structures. Biswas and Nanda (1994) investigated the roughness in groups and in subgroups. Kuroki was the first who applied the concept of roughness in semigroups (Kuroki, 1997), while Davvaz (2004) studied roughness in rings. Other researchers investigated roughness in numerous algebraic structures (Davvaz & Mahdavipour, 2006; Davvaz, 2008; Jun, 2008; Sun & Ma, 2014; Sun *et al*., 2016). Hosseini has applied generalised rough set in fuzzy algebraic structures (Hosseini, 2011; Hosseini *et al*., 2012). However, in the case of $(\in, \in \vee q)$-fuzzy algebraic structures much attention has not been paid. Therefore, it is important to study the roughness in generalised fuzzy algebraic structures such as $(\in, \in \vee q)$-fuzzy algebraic structures.

METHODOLOGY

This section deals with some basic concepts of LA-semigroups and their ideals; fuzzy set, fuzzy ideals, $(\in, \in \vee q)$-fuzzy ideals of LA-semigroups, and different types of set valued homomorphism. These notions will be helpful in later sections.

A groupoid $(S, *)$ is called an LA-semigroup, if it satisfies the left invertive law

$$(a * b) * c = (c * b) * a \quad \text{for all } a, b, c \in S.$$

Throughout the paper S and R will denote LA-semigroups. Let S be an LA-semigroup and A be a subset of S. Then A is called an LA-subsemigroup of S, if $A^2 \subseteq A$, that is $ab \in A$ for all $a, b \in A$. A subset A of an LA-semigroup S is called left ideal (right ideal) of S, if $SA \subseteq A\,(AS \subseteq A)$. An LA-subsemigroup A of an LA-semigroup S is called bi-ideal of S, if $(AS)\,A \subseteq A$. An LA-subsemigrup A of an LA-semigroup S is called an interior ideal of S, if $(SA)\,S \subseteq A$. An element a of LA-semigroup S is called idempotent, if $aa = a$. If every element of S is idempotent, then S is called idempotent.

A fuzzy subset μ of an LA-semigroup S is a function $\mu : S \to [0,1]$. A fuzzy subset μ of an LA-semigroup S is called a fuzzy LA-subsemigroup of S, if $\mu(ab) \geq \mu(a) \wedge \mu(b)$ for all $a, b \in S$. A fuzzy subset μ of an LA-semigroup S is called a fuzzy left (fuzzy right) ideal of S, if $\mu(ab) \geq \mu(b)\,(\mu(ab) \geq \mu(a))$ for all $a, b \in S$. A fuzzy LA-subsemigroup μ of an LA- semigroup S is called fuzzy interior ideal of S, if $\mu((xa)\,y) \geq \mu(a)$ for all a, x and $y \in S$. Let μ be a fuzzy LA-subsemigroup of an LA-semigroup S. Then μ is fuzzy bi-ideal of S, if $\mu((xa)\,y) \geq \mu(x) \wedge \mu(y)$ for all x, y and $a \in S$.

Definition 1. Let μ and δ be fuzzy LA-subsemigroups of an LA-semigroup S. Then their composition is defined as

$$(\mu \circ \delta)(y) = \begin{cases} \bigvee_{y=ab} \{\mu(a) \wedge \delta(b)\} & \text{if } y = ab \text{ for } a, b \in S \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2. (Khan *et al*., 2010a; 2010b) A fuzzy subset μ of an LA-semigroup S of the form

$$\mu(x) = \begin{cases} t \in (0,1] & \text{if } x = y \\ 0 & \text{otherwise} \end{cases}$$

is said to be a fuzzy point with support x and value t and is denoted by x_t.

Definition 3. (Khan *et al*., 2010a; 2010b) A fuzzy point x_t is said to be 'belongs to' (resp.'quasi-coincident with') a fuzzy subset μ written $x_t \in \mu\,(\text{resp. } x_t q \mu)$ if $\mu(x) \geq t\ (\text{resp. } \mu(x) + t > 1)$.

Definition 4. (Khan *et al*., 2010a; 2010b) Let μ be a fuzzy subset of an LA-semigroup S. Then μ is called an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of S if the following condition holds:

$$(\forall x, y \in S)\,(\forall t_1, t_2 \in (0,1])$$

$$\left(x_{t_1}, y_{t_2} \in \mu \to (xy)_{\min\{t_1,t_2\}} \in \vee q \mu\right).$$

Theorem 1. (Khan *et al*., 2010a; 2010b) Let μ be a fuzzy subset of an LA-semigroup S. Then μ is an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of S if and only if $\mu(xy) \geq \min\{\mu(x), \mu(y), 0.5\}$ for all $x, y \in S$.

Definition 5. (Khan *et al*., 2010a; 2010b) Let μ be a fuzzy subset of an LA-semigroup S. Then μ is called an $(\in, \in \vee q)$- fuzzy left ideal of S if the following condition holds:

$$(\forall x, y \in S)\,(\forall t \in (0,1])\,(y_t \in \mu \to (xy)_t \in \vee q \mu).$$

The $(\in, \in \vee q)$-fuzzy right ideal can be defined analogously.

Theorem 2. (Khan *et al*., 2010a; 2010b) Let μ be a fuzzy subset of an LA-semigroup *S*. Then μ is an $(\in, \in \vee q)$-fuzzy left ideal of *S* if and only if $\mu(ab) \geq \min\{\mu(b), 0.5\}$ for all $a, b \in S$.

Definition 6. (Khan *et al*., 2010a; 2010b) Let μ be an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of an LA-semigroup *S*. Then μ is called an $(\in, \in \vee q)$-fuzzy bi-ideal of *S* if the following condition holds:

$$(\forall x, a, y \in S)\,(\forall t_1, t_2 \in (0,1])$$

$$\left(x_{t_1}, y_{t_2} \in \mu \rightarrow ((xa)\,y)_{\min\{t_1,t_2\}} \in \vee q\mu\right).$$

Theorem 3. (Khan *et al*., 2010a; 2010b) Let μ be an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of an LA-semigroup *S*. Then μ is an $(\in, \in \vee q)$-fuzzy bi-ideal of *S* if and only if $\mu((xa)y) \geq \min\{\mu(x), \mu(y), 0.5\}$ for all $x, y \in S$.

Definition 7. (Khan *et al*., 2010a; 2010b) Let μ be an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of an LA-semigroup *S*. Then μ is called an $(\in, \in \vee q)$-fuzzy interior ideal of *S* if the following condition holds:

$$(\forall x, a, y \in S)\,(\forall t \in (0,1])\,(a_t \in \mu \rightarrow ((xa)\,y)_t \in \vee q\mu).$$

Theorem 4. (Khan *et al*., 2010a; 2010b) Let μ be an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of an LA-semigroup *S*. Then μ is an $(\in, \in \vee q)$-fuzzy interior ideal in *S* if and only if $\mu((xa)y) \geq \min\{\mu(a), 0.5\}$ for all a, y and $a \in S$.

Definition 8. Let *R* and *S* be two LA-semigroups and $T : R \rightarrow P(S)$ be a set valued (SV) mapping. Then *T* is called an SV-homomorphism, if $T(a)\,T(b) \subseteq T(ab)$ for all $a, b \in R$.

Definition 9. Let $T : S \;\; P(S)$ be an SV-homomorphism. Then *T* is called reflexive if $a \in T(a)$ for all $a \in S$. In this paper reflexive set valued homomorphism will be denoted by RSV-homomorphism.

Example 1. Let $R = \{a, b, c\}$ be an LA-semigroup where multiplication is defined by Table 1:

Table 1: LA-semigroup R

.	*a*	*b*	*c*
a	*a*	*a*	*a*
b	*c*	*c*	*c*
c	*a*	*a*	*c*

Define an SV-mapping $T : R \rightarrow P(R)$ by $T(a) = T(c) = \{a, b, c\}$ and $T(b) = \{b, c\}$. Then *T* is an RSV-homomorphism.

Example 2. Let $S = \{a, b, c, d, e\}$ be an LA-semigroup with the multiplication Table 2:

Table 2: LA-semigroup S

.	*a*	*b*	*c*	*d*	*e*
a	*d*	*d*	*d*	*d*	*d*
b	*d*	*d*	*d*	*d*	*d*
c	*d*	*d*	*d*	*d*	*d*
d	*d*	*d*	*d*	*d*	*d*
e	*d*	*d*	*c*	*d*	*d*

Define an SV-mapping $T: S \rightarrow P(S)$ by $T(a) = \{a\}$, $T(b) = \{b\}$, $T(c) = T(d) = \{c, d\}$ and $T(e) = \{c\}$. Then *T* is an SV-homomorphism.

Definition 10. Let *R* and *S* be two LA-semigroups and $T : R \rightarrow P(S)$ be a set valued mapping. Then *T* is called a strong set valued (SSV) homomorphism, if $T(a)T(b) = T(ab)$ for all $a, b \in R$.

Example 3. Let $R = \{a, b, c\}$ and $S = \{1, 2, 3\}$ be two LA-semigroups with multiplication tables as given by Tables 3 and 4, respectively:

Table 3: LA-semigroup R

.	*a*	*b*	*c*
a	*b*	*c*	*b*
b	*c*	*c*	*c*
c	*c*	*c*	*c*

Table 4: LA-semigroup *S*

.	1	2	3
1	2	3	2
2	3	3	3
3	3	3	3

Define an SV-mapping $T: R \rightarrow P(S)$ by $T(a) = T(c) = \{3\}$ and $T(b) = \{2,3\}$. Then *T* is an *SSV*-homomorphism.

RESULTS AND DISCUSSION

This section deals with generalised roughness in fuzzy sets and the approximations of $(\in, \in \vee q)$-fuzzy LA-subsemigroups. An example is provided to show that the lower approximation of an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of an LA-semigroup *S* is not an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of *S* under an SV-homomorphism.

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Definition 11. Let $T : S \rightarrow P(S)$ be an SV-mapping. Let μ be a fuzzy subset of *S*. For every *S*, define T-rough lower and T-rough upper fuzzy approximations of μ by

$$\underline{T}(\mu)(x) = \bigwedge_{a \in T(x)} \mu(a)$$

and

$$\overline{T}(\mu)(x) = \bigvee_{a \in T(x)} \mu(a).$$

A fuzzy subset μ is fuzzy definable if $\underline{T}(\mu)(x) = \overline{T}(\mu)(x)$ for all $x \in S$. Otherwise μ is a fuzzy rough set.

In the following proposition it is seen that the upper approximation of an $(\in, \in \vee q)$-fuzzy LA-subsemigroup is an $(\in, \in \vee q)$-fuzzy LA-subsemigroup.

Proposition 1. Let $T : S \rightarrow P(S)$ be an SV-homomorphism and let μ be a fuzzy subset of *S*. If μ is an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of *S*, then *T(μ) s* an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of *S*.

Proof. Let *a, b* $\in S$ and μ be an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of *S*. Suppose $a_{t_1}, b_{t_2} \in \overline{T}(\mu)$. Then $\overline{T}(\mu)(a) \geq t_1$ and $\overline{T}(\mu)(b) \geq t_2$, where $t_1, t_2 \in (0, 1]$. It follows that

$$\begin{aligned}
\min\{t_1, t_2, 0.5\} &\leq \overline{T}(\mu)(a) \wedge \overline{T}(\mu)(b) \wedge 0.5 \\
&= \bigvee_{x \in T(a)} (\mu(x)) \wedge \bigvee_{y \in T(b)} (\mu(y)) \wedge 0.5 \\
&= \bigvee_{x \in T(a), y \in T(b)} (\mu(x) \wedge \mu(y) \wedge 0.5) \\
&\leq \bigvee_{x \in T(a), y \in T(b)} \mu(xy) \\
&= \left(\bigvee_{xy \in T(a)T(b)} \mu(xy) \right) \\
&= \left(\bigvee_{z \in T(a)T(b)} \mu(z) \right) \qquad (\text{where } z = xy) \\
&\leq \left(\bigvee_{z \in T(ab)} \mu(z) \right) \\
&= \overline{T}(\mu)(ab).
\end{aligned}$$

This implies that $\min\{t_1, t_2, 0.5\} \leq \overline{T}(\mu)(ab)$.

Hence by Theorem 1 $\overline{T}(\mu)$ is an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of S. □

In the following example it is shown that if T is an SV-homomorphism then for an $(\in, \in \vee q)$ fuzzy LA-subsemigroup its lower approximation $\underline{T}(\mu)$ may not be an $(\in, \in \vee q)$-fuzzy LA-subsemigroup.

Example 4. Consider the LA-semigroup *S* of Eample 2. Define an SV-mapping $T : S \rightarrow P(S)$ by $T(a) = \{a\}$, $T(b) = \{b\}$, $T(c) = T(d) = \{c,d\}$ and $T(e) = \{c\}$. Then T is an SV-homomorphism. Let μ be a fuzzy subset of *S* defined by $\mu(a) = 0.5$, $\mu(b) = 0.6$, $\mu(c) = 0.44$, $\mu(d) = 0.6$ and $\mu(e) = 0.55$. Then $\underline{T}(\mu)(a) = 0.5$, $\underline{T}(\mu)(b) = 0.6$, $\underline{T}(\mu)(c) = \mathrm{T}(\mu)(d) = 0.44$ and $\underline{T}(\mu)(e) = 0.6$. Let $a_{0.5}, b_{0.5}, c_{0.44}, d_{0.44}, e_{0.45} \in \underline{T}(\mu)$. Clearly μ is an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of *S*. Now $e_{0.45} \in \underline{T}(\mu)$ but $(ee)_{\min\{0.45, 0.45\}} = (d)_{0.45} \overline{\in} \underline{T}(\mu)$, since $\underline{T}(\mu)(d) = 0.44 \ngeq 0.45$. Also $\underline{T}(\mu)(ee) + \min\{0.45, 0.45\} \ngtr 1$.

Which implies that $(ee)_{\min\{0.45, 0.45\}} \overline{q} \underline{T}(\mu)$. Hence $\underline{T}(\mu)$ is not an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of *S*.

However the following can be established.

Proposition 2. Let $T : S \rightarrow P(S)$ be an SSV-homomorphism and μ be a fuzzy subset of *S*. If μ is an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of *S*, then $\underline{T}(\mu)$ is an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of *S*.

Proof. Let *a*, *b* $\in S$ and μ be an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of *S*. Suppose $a_{t_1}, b_{t_2} \in \underline{T}(\mu)$. Then $\underline{T}(\mu)(a) \geq t_1$ and $\underline{T}(\mu)(b) \geq t_2$, where $t_1, t_2 \in (0, 1]$. It follows that

$$\begin{aligned}
\underline{T}(\mu)(ab) &= \bigwedge_{z \in T(ab)} \mu(z) \\
&= \bigwedge_{z \in T(a)T(b)} \mu(z) \\
&= \bigwedge_{xy \in T(a)T(b)} \mu(xy) \quad \left(\begin{array}{c} \text{where } z = xy \text{ for some} \\ x \in T(a) \text{ and } y \in T(b) \end{array} \right) \\
&\geq \bigwedge_{x \in T(a), y \in T(b)} (\mu(x) \wedge \mu(y) \wedge 0.5) \\
&= \left(\bigwedge_{x \in T(a)} (\mu(x)) \right) \wedge \left(\bigwedge_{y \in T(b)} (\mu(y)) \right) \wedge 0.5 \\
&= \underline{T}(\mu)(a) \wedge \underline{T}(\mu)(b) \wedge 0.5 \\
&\geq \min\{t_1, t_2, 0.5\}.
\end{aligned}$$

This implies that $\underline{T}(\mu)(ab) \geq \min\{t_1, t_2, 0.5\}$. Therefore, by Theorem 1, $\underline{T}(\mu)$ is an $(\in, \in \vee q)$-fuzzy LA-subsemigroup of S. □

Approximations of lower and upper parts of $(\in, \in \vee q)$-fuzzy subset of LA-semigroups

Notion of lower and upper part of a fuzzy set is given in Shabir *et al*. (2010). In the following this notion is defined for $(\in, \in \vee q)$-fuzzy sets on LA-semigroups. In this section some properties of lower and upper approximations of lower and upper parts of $(\in, \in \vee q)$-fuzzy sets have been studied.

Definition 12. Let μ be a fuzzy subset of an LA-semigroup *S*. Define the upper and lower parts of μ respectively as follows:

$$\mu^{+}(x) = \mu(x) \vee 0.5$$

and

$$\mu^{-}(x) = \mu(x) \wedge 0.5.$$

Proposition 3. Let $T : S \to P(S)$ be an SV-homomorphism. If μ and δ are fuzzy subsets of *S*, then the following hold:

(i) $\overline{T}(\mu \wedge \delta)^{-} = \overline{T}(\mu)^{-} \wedge \overline{T}(\delta)^{-}$
(ii) $\overline{T}(\mu \wedge \delta)^{+} = \overline{T}(\mu)^{+} \wedge \overline{T}(\delta)^{+}$
(iii) $\overline{T}(\mu \vee \delta)^{-} = \overline{T}(\mu)^{-} \vee \overline{T}(\delta)^{-}$
(iv) $\overline{T}(\mu \vee \delta)^{+} = \overline{T}(\mu)^{+} \vee \overline{T}(\delta)^{+}$

Proof. (i) Let $y \in S$. Then

$$\begin{aligned}
\overline{T}(\mu \wedge \delta)^{-}(y) &= \bigvee_{a \in T(y)} (\mu \wedge \delta)^{-}(a) \\
&= \bigvee_{a \in T(y)} ((\mu \wedge \delta)(a) \wedge 0.5) \\
&= \bigvee_{a \in T(y)} (\mu(a) \wedge \delta(a) \wedge 0.5) \\
&= \bigvee_{a \in T(y)} (\mu(a) \wedge 0.5) \wedge \bigvee_{a \in T(y)} (\delta(a) \wedge 0.5) \\
&= \bigvee_{a \in T(y)} (\mu^{-}(a)) \wedge \bigvee_{a \in T(y)} (\delta^{-}(a)) \\
&= \overline{T}(\mu^{-})(y) \wedge \overline{T}(\delta^{-})(y).
\end{aligned}$$

Hence $\overline{T}(\mu \wedge \delta)^{-} = \overline{T}(\mu)^{-} \wedge \overline{T}(\delta)^{-}$.

(ii) Let $y \in S$. Then

$$\begin{aligned}
\overline{T}(\mu \wedge \delta)^{+}(y) &= \bigvee_{a \in T(y)} (\mu \wedge \delta)^{+}(a) \\
&= \bigvee_{a \in T(y)} ((\mu \wedge \delta)(a) \vee 0.5) \\
&= \bigvee_{a \in T(y)} ((\mu(a) \wedge \delta(a)) \vee 0.5) \\
&= \bigvee_{a \in T(y)} (\mu(a) \vee 0.5) \wedge \bigvee_{a \in T(y)} (\delta(a) \vee 0.5) \\
&= \bigvee_{a \in T(y)} (\mu^{+}(a)) \wedge \bigvee_{a \in T(y)} (\delta^{+}(a)) \\
&= \overline{T}(\mu^{+})(y) \wedge \overline{T}(\delta^{+})(y)
\end{aligned}$$

Hence $\overline{T}(\mu \wedge \delta)^{+} = \overline{T}(\mu)^{+} \wedge \overline{T}(\delta)^{+}$.

(iii) Proof follows from (i) and Definition 12.
(iv) Proof follows from (ii) and Definition 12.

Proposition 4. Let $T : S \to P(S)$ be an SV-homomorphism. If μ and δ are fuzzy subsets of *S* then the following hold:

(i) $\underline{T}(\mu \wedge \delta)^{-} = \underline{T}(\mu)^{-} \wedge \underline{T}(\delta)^{-}$
(ii) $\underline{T}(\mu \wedge \delta)^{+} = \underline{T}(\mu)^{+} \wedge \underline{T}(\delta)^{+}$
(iii) $\underline{T}(\mu \vee \delta)^{-} = \underline{T}(\mu)^{-} \vee \underline{T}(\delta)^{-}$
(iv) $\underline{T}(\mu \vee \delta)^{+} = \underline{T}(\mu)^{+} \vee \underline{T}(\delta)^{+}$

Proof. (i) Let $x \in S$. Then

$$\begin{aligned}
\underline{T}(\mu \wedge \delta)^{-}(y) &= \bigwedge_{a \in T(y)} (\mu \wedge \delta)^{-}(a) \\
&= \bigwedge_{a \in T(y)} ((\mu \wedge \delta)(a) \wedge 0.5) \\
&= \bigwedge_{a \in T(y)} ((\mu(a) \wedge \delta(a)) \wedge 0.5) \\
&= \bigwedge_{a \in T(y)} (\mu(a) \wedge 0.5) \wedge \bigwedge_{a \in T(y)} (\delta(a) \wedge 0.5) \\
&= \bigwedge_{a \in T(y)} (\mu^{-}(a)) \wedge \bigwedge_{a \in T(y)} (\delta^{-}(a)) \\
&= \underline{T}(\mu^{-})(y) \wedge \underline{T}(\delta^{-})(y)
\end{aligned}$$

Hence $\underline{T}(\mu \wedge \delta)^{-} = \underline{T}(\mu)^{-} \wedge \underline{T}(\delta)^{-}$.

(ii) Let $y \in S$. Then

$$\begin{aligned}
\underline{T}(\mu \wedge \delta)^{+}(y) &= \bigwedge_{a \in T(y)} (\mu \wedge \delta)^{+}(a) \\
&= \bigwedge_{a \in T(y)} ((\mu \wedge \delta)(a) \vee 0.5) \\
&= \bigwedge_{a \in T(y)} ((\mu(a) \wedge \delta(a)) \vee 0.5) \\
&= \bigwedge_{a \in T(y)} (\mu(a) \vee 0.5) \wedge \bigwedge_{a \in T(y)} (\delta(a) \vee 0.5) \\
&= \bigwedge_{a \in T(y)} (\mu^{+}(a)) \wedge \bigwedge_{a \in T(x)} (\delta^{+}(a)) \\
&= \underline{T}(\mu^{+})(y) \wedge \underline{T}(\delta^{+})(y)
\end{aligned}$$

Hence $\underline{T}(\mu \wedge \delta)^{+} = \underline{T}(\mu)^{+} \wedge \underline{T}(\delta)^{+}$.

(iii) Proof follows from (i) and Definition 12.
(iv) Proof follows from (i) and Definition 12. □

Proposition 5. Let *S* be an LA-semigroup and $T : S \to P(S)$ be an SV-homomorphism. If μ and δ are fuzzy ideals of *S*, then

$$\overline{T}(\mu \circ \delta)^{-}(y) \leq \overline{T}(\mu)^{-}(y) \wedge \overline{T}(\delta)^{-}(y).$$

Proof. Let μ and δ be fuzzy ideals of S. Then

$$\begin{aligned}(\mu\circ\delta)(y) &\leq (\mu\circ S)(y)\\ &= \bigvee_{y=ab}\{\mu(a)\wedge S(b)\}\\ &= \bigvee_{y=ab}\{\mu(a)\wedge 1\}\\ &= \mu(a)\\ &\leq \mu(ab)=\mu(y).\end{aligned}$$

That is $(\mu\circ\delta)(y)\leq\mu(y)$. Now $(\mu\circ\delta)(y)\wedge 0.5\leq\mu(y)\wedge 0.5$, which implies $(\mu\circ\delta)^{-}(y)\leq\mu^{-}(y)$. Similarly it can be shown that $(\mu\circ\delta)^{-}(y)\leq\delta^{-}(y)$. Hence $\overline{T}(\mu\circ\delta)^{-}(y)\leq\overline{T}(\mu)^{-}(y)$ and $\overline{T}(\mu\circ\delta)^{-}(y)\leq\overline{T}(\delta)^{-}(y)$. Therefore $\overline{T}(\mu\circ\delta)^{-}(y)\leq\overline{T}(\mu)^{-}(y)\wedge\overline{T}(\delta)^{-}(y)$. □

In general equality does not hold in Proposition 5. The following example makes the situation clear.

Example 5. Consider the LA-semigroup S of Example 2. Define an SV-mapping $T:S\rightarrow P(S)$ by $T(a)=\{a\}$, $T(b)=\{b\}$, $T(c)=T(d)=\{c,d\}$ and $T(e)=\{c\}$. Then T is an SV-homomorphism. Let μ,δ be fuzzy subsets of S defined by $\mu(a)=\mu(b)=\mu(e)=0.2$, $\mu(d)=0.4$, $\mu(c)=0.35$ and $\delta(a)=\delta(b)=\delta(e)=0.1$, $\delta(c)=0.3$, $\delta(d)=0.38$. Then clearly μ and δ are fuzzy ideals of S. Also $(\mu\circ\delta)^{-}(a)=(\mu\circ\delta)^{-}(b)=(\mu\circ\delta)^{-}(e)=0,(\mu\circ\delta)^{-}(c)=0.2,(\mu\circ\delta)^{-}(d)=0.38$. So $\overline{T}(\mu\circ\delta)^{-}(a)=0$. *Also* $\overline{T}(\mu)^{-}(a)=0.2,\overline{T}(\delta)^{-}(a)=0.1$. This implies that $\overline{T}(\mu\circ\delta)^{-}(a)\ngeq\left\{\overline{T}(\mu)^{-}(a)\wedge\overline{T}(\delta)^{-}(a)\right\}$.

However in case of an idempotent LA-semigroup equality can be shown.

Proposition 6. Let S be an idempotent LA- semigroup and T:S→$P(S)$ be an SV homomorphism. If μ and δ are fuzzy ideals of S, then

$$\overline{T}(\mu\circ\delta)^{-}(y)=\overline{T}(\mu)^{-}(y)\wedge\overline{T}(\delta)^{-}(y).$$

Proof. From Proposition 5, it is obvious that $\overline{T}(\mu\circ\delta)^{-}(y)\leq\overline{T}(\mu)^{-}(y)\wedge\overline{T}(\delta)^{-}(y)$. For reverse inequality, let $y\in S$. It follows that;

$$\begin{aligned}\overline{T}(\mu)^{-}(y)\wedge\overline{T}(\delta)^{-}(y) &= \left(\bigvee_{a\in T(y)}(\mu^{-}(a))\right)\wedge\left(\bigvee_{b\in T(y)}(\delta^{-}(b))\right)\\ &= \bigvee_{a\in T(y),b\in T(y)}(\mu^{-}(a)\wedge\delta^{-}(b))\\ &= \bigvee_{ab\in T(y)T(y)}((\mu(a)\wedge 0.5)\wedge(\delta(b)\wedge 0.5))\\ &\leq \bigvee_{ab\in T(yy)}((\mu(a)\wedge\delta(b))\wedge 0.5)\\ &= \bigvee_{ab\in T(y)}((\mu(a)\wedge\delta(b))\wedge 0.5)\\ &= \bigvee_{z\in T(y)}\left(\bigvee_{z=ab}(\mu(a)\wedge\delta(b))\wedge 0.5\right)\\ &= \bigvee_{z\in T(y)}((\mu\circ\delta)(z)\wedge 0.5)\\ &= \bigvee_{z\in T(y)}(\mu\circ\delta)^{-}(z)\\ &= \overline{T}(\mu\circ\delta)^{-}(y).\end{aligned}$$

This implies that $\overline{T}(\mu)^{-}(y)\wedge\overline{T}(\delta)^{-}(y)\leq\overline{T}(\mu\circ\delta)^{-}(y)$. Hence $\overline{T}(\mu\circ\delta)^{-}(y)=\overline{T}(\mu)^{-}(y)\wedge\overline{T}(\delta)^{-}(y)$. □

Proposition 7. Let S be an idempotent LA-semigroup and $T:S\rightarrow P(S)$ be an SSV homomorphism. If μ and δ are fuzzy ideals of S, then

$$\underline{T}(\mu\circ\delta)^{-}(y)=\underline{T}(\mu)^{-}(y)\wedge\underline{T}(\delta)^{-}(y).$$

Proof. The proof of this Proposition is similar to Proposition 6. □

Remark 1. It is felt that in Propositions 6 and 7, idempotency of S is a strong condition, so there is a question. Can we relax it by some weaker condition?

Generalised roughness in $(\in,\in\vee q)$-fuzzy ideals

In this subsection some properties of lower and upper approximations for fuzzy ideals of LA-semigroups are studied.

Proposition 8. Let $T:S\rightarrow P(S)$ be an RSV-homomorphism. If μ is an $(\in,\in\vee q)$-fuzzy left ideal of S, then $\overline{T}(\mu)$ is an $(\in,\in\vee q)$-fuzzy left ideal of S.

Proof. Let $a,b\in S$ and μ be an $(\in,\in\vee q)$-fuzzy left ideal of S: Suppose $b_t\in\overline{T}(\mu)$. Then $\overline{T}(\mu)(b)\geq t$ where $t\in(0,1]$. It follows that

$$\begin{aligned}\min\{t,0.5\} &\leq \overline{T}(\mu)(b)\wedge 0.5\\ &= \bigvee_{y\in T(b)}(\mu(y))\wedge 0.5\\ &= \bigvee_{y\in T(b)}(\mu(y)\wedge 0.5)\\ &\leq \bigvee_{a\in T(a),y\in T(b)}\mu(ay)\\ &= \left(\bigvee_{ay\in T(a)T(b)}\mu(ay)\right)\end{aligned}$$

$$\begin{aligned} &= \left(\bigvee_{z\in T(a)T(b)} \mu(z)\right) \quad (\text{where } z = ay) \\ &\leq \left(\bigvee_{z\in T(ab)} \mu(z)\right) \\ &= \overline{T}(\mu)(ab). \end{aligned}$$

This implies that min $\{t_1, t_2, 0.5\} \leq \overline{T}(\mu)(ab)$.

Therefore, by Theorem 2, $\overline{T}(\mu)$ is an $(\in, \in \vee q)$-fuzzy left ideal of *S*. □

Proposition 9. Let $T : S \to P(S)$ be an *SSV*-homomorphism. If μ is an $(\in, \in \vee q)$-fuzzy left ideal of *S*, then $\underline{T}(\mu)$ is an $(\in, \in \vee q)$-fuzzy left ideal of *S*:

Proof. Let $a, b \in S$ and μ be an $(\in, \in \vee q)$-fuzzy left ideal of *S*. Suppose $b_t \in \underline{T}(\mu)$. Then $\underline{T}(\mu)(b) \geq t$, where $t \in (0,1]$. It follows that

$$\begin{aligned} \underline{T}(\mu)(ab) &= \bigwedge_{z\in T(ab)} \mu(z) \\ &= \bigwedge_{z\in T(a)T(b)} \mu(z) \\ &= \bigwedge_{xy\in T(a)T(b)} \mu(xy) \quad \left(\begin{array}{c}\text{where } z = xy \text{ for some} \\ x \in T(a) \text{ and } y \in T(b)\end{array}\right) \\ &= \bigwedge_{x\in T(a), y\in T(b)} \mu(xy) \\ &\geq \bigwedge_{y\in T(b)} (\mu(y) \wedge 0.5) \\ &= \bigwedge_{y\in T(b)} \mu(y) \wedge 0.5 \\ &= \underline{T}(\mu)(b) \wedge 0.5 \\ &\geq \{t \wedge 0.5\}. \end{aligned}$$

This implies that $\underline{T}(\mu)(ab) \geq \{t \wedge 0.5\}$. So by Theorem 2 $\underline{T}(\mu)$ is an $(\in, \in \vee q)$-fuzzy left ideal of *S*. □

Proposition 10. Let $T : S \to P(S)$ be an *RSV*-homomorphism. If μ is an $(\in, \in \vee q)$-fuzzy interior ideal of *S*, then $\overline{T}(\mu)$ is an $(\in, \in \vee q)$-fuzzy interior ideal of *S*.

Proof. Let $x, y, a \in S$ and let μ be an $(\in, \in \vee q)$-fuzzy interior ideal of *S*. Let $a_t \in \overline{T}(\mu)$. Then $\overline{T}(\mu)(a) \geq t$, where $t \in (0,1]$. It follows that

$$\begin{aligned} \min\{t, 0.5\} &\leq \overline{T}(\mu)(a) \wedge 0.5 \\ &= \bigvee_{c\in T(a)} \mu(c) \wedge 0.5 \\ &\leq \bigvee_{x\in T(x), c\in T(a), y\in T(y)} \mu((xc)y) \\ &= \bigvee_{xc\in T(x)T(a), y\in T(y)} \mu((xc)y) \\ &\leq \bigvee_{xc\in T(xa), y\in T(y)} \mu((xc)y) \\ &= \bigvee_{(xc)y\in T(xa)T(y)} \mu((xc)y) \\ &\leq \bigvee_{(xc)y\in T((xa)y)} \mu((xc)y) \\ &= \bigvee_{z\in T((xa)y)} \mu(z) \quad (\text{where } z = (xc)y) \\ &= \overline{T}(\mu)((xa)y). \end{aligned}$$

This implies that $\min\{t, 0.5\} \leq \overline{T}(\mu)((xa)y)$. Therefore by Theorem 4, $\overline{T}(\mu)$ is an $(\in, \in \vee q)$-fuzzy interior ideal of *S*.

Proposition 11. Let $T : S \to P(S)$ be an *SSV*-homomorphism. If μ is an $(\in, \in \vee q)$-fuzzy interior ideal of S, then $\overline{T}(\mu)$ is an $(\in, \in \vee q)$-fuzzy interior ideal of *S*.

Proof. Let $x, y, a \in S$ and μ be an $(\in, \in \vee q)$-fuzzy interior ideal of *S*. Let $a_t \in \underline{T}(\mu)$. Then $\underline{T}(\mu)(a) \geq t$, where $t \in (0,1]$. It follows that

$$\begin{aligned} \underline{T}(\mu)((xa)y) &= \bigwedge_{z\in T((xa)y)} \mu(z) \\ &= \bigwedge_{z\in T(xa)T(y)} \mu(z) \\ &= \bigwedge_{(bc)d\in T(xa)T(y)} \mu((bc)d) \quad (\text{ where } z = (bc)d) \\ &= \bigwedge_{bc\in T(xa), d\in T(y)} \mu((bc)d) \\ &= \bigwedge_{bc\in T(x)T(a), d\in T(y)} \mu((bc)d) \\ &= \bigwedge_{b\in T(x), c\in T(a), d\in T(y)} \mu((bc)d) \\ &\geq \bigwedge_{c\in T(a)} (\mu(c) \wedge 0.5) \\ &= \bigwedge_{c\in T(a)} \mu(c) \wedge 0.5 \\ &= \underline{T}(\mu)(a) \wedge 0.5 \\ &\geq \min\{t, 0.5\}. \end{aligned}$$

This implies that $\underline{T}(\mu)((xa)y) \geq \min\{t, 0.5\}$. Hence by Theorem 4, $\underline{T}(\mu)$ is an $(\in, \in \vee q)$-fuzzy interior ideal of *S*. □

Proposition 12. Let $T : S \to P(S)$ be an *RSV*-homomorphism. If μ is an $(\in, \in \vee q)$-fuzzy bi-ideal of *S*, then $\underline{T}(\mu)$ is an $(\in, \in \vee q)$-fuzzy bi-ideal of *S*:

Proof. Let $x, y, a \in S$ and μ be an $(\in, \in \vee q)$-fuzzy bi-ideal of *S*: Let $x_{t_s}, y_{t_p} \in \overline{T}(\mu)$. Then $\overline{T}(\mu)(x) \geq t_s$ and $\overline{T}(\mu)(y) \geq t_p$ where $t_s, t_p \in (0,1]$. It follows that

$$
\begin{aligned}
\min\{t_s,t_p,0.5\} &\leq \overline{T}(\mu)(x)\wedge\overline{T}(\mu)(y)\wedge 0.5\\
&= \bigvee_{b\in T(x)}\mu(b)\wedge\bigvee_{d\in T(y)}\mu(d)\wedge 0.5\\
&= \bigvee_{b\in T(x),d\in T(y)}(\mu(b)\wedge\mu(d)\wedge 0.5)\\
&\leq \bigvee_{b\in T(x),a\in T(a),d\in T(y)}\mu((ba)d)\\
&= \bigvee_{ba\in T(x)T(a),d\in T(y)}\mu((ba)d)\\
&\leq \bigvee_{ba\in T(xa),d\in T(y)}\mu((ba)d)\\
&= \bigvee_{(ba)d\in T(xa)T(y)}\mu((ba)d)\\
&\leq \bigvee_{(ba)d\in T((xa)y)}\alpha((ba)d)\\
&= \bigvee_{z\in T((xa)y)}\mu(z) \qquad (\text{where } z=(ba)d)\\
&= \overline{T}(\mu)((xa)y).
\end{aligned}
$$

This implies that $\min\{t_s,t_p,0.5\}\leq\overline{T}(\mu)((xa)y)$. Hence by Theorem 3, $((xa)y)_{\min\{t_s,t_p\}}\in\vee q\overline{T}(\mu)$. Therefore $\overline{T}(\mu)$ is an $(\in,\in\vee q)$-bi-ideal of *S*:

Proposition 13. Let $T : S \rightarrow P(S)$ be an *SSV*-homomorphism. If μ is an $(\in,\in\vee q)$-fuzzy bi-ideal of *S*; then $\overline{T}(\mu)$ is an $(\in,\in\vee q)$-fuzzy bi-ideal of *S*.

Proof. Let $x,y,a\in S$ and μ be an $(\in,\in\vee q)$-fuzzy bi-ideal of *S*. Let $x_{t_s},y_{t_p}\in\underline{T}(\mu)$. Then $\underline{T}(\mu)(x)\geq t_s$ and $\underline{T}(\mu)(y)\geq t_p$, where $t_s,t_p\in(0,1]$. It follows that

$$
\begin{aligned}
\underline{T}(\mu)((xa)y) &= \bigwedge_{z\in T((xa)y)}\mu(z)\\
&= \bigwedge_{z\in T(xa)T(y)}\mu(z)\\
&= \bigwedge_{(bc)d\in T(xa)T(y)}\mu((bc)d) \qquad (\text{where } z=(bc)d)\\
&= \bigwedge_{bc\in T(xa),d\in T(y)}\mu((bc)d)\\
&= \bigwedge_{bc\in T(x)T(a),d\in T(y)}\mu((bc)d)\\
&= \bigwedge_{b\in T(x),c\in T(a),d\in T(y)}\mu((bc)d)\\
&\geq \bigwedge_{b\in T(x),d\in T(y)}(\mu(b)\wedge\mu(d)\wedge 0.5)\\
&= \bigwedge_{b\in T(x)}\mu(b)\wedge\bigwedge_{d\in T(y)}\mu(d)\wedge 0.5\\
&= \underline{T}(\mu)(x)\wedge\underline{T}(\mu)(y)\wedge 0.5\\
&\geq \min\{t_1,t_2,0.5\}.
\end{aligned}
$$

This implies that $\underline{T}(\mu)((xa)y)\geq\min\{t_s,t_p,0.5\}$. Thus $((xa)y)_{\min\{t_s,t_p\}}\in\vee q\underline{T}(\mu)$. Therefore by Theorem 3, $\overline{T}(\mu)$ is an $(\in,\in\vee q)$-fuzzy bi-ideal of *S*:

CONCLUSION

Non associative algebras are being studied all over the globe, in particular LA-semigroups have attracted many authors and researchers. The $(\in,\in\vee q)$-fuzzy algebraic substructures are generalisations of fuzzy algebraic sub-structures. In this paper, generalised roughness has been studied for $(\in,\in\vee q)$-fuzzy algebraic substructures of LA-semigroups. It is observed that, in order to preserve a particular algebraic substructure in case of its approximations, many types of set valued homomorphisms are required.

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