

## RESEARCH ARTICLE

# $\alpha_*$ -dominated set-valued mappings and some generalised fixed point results

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**Abstract:** In this paper, we established some fixed point results for a pair of  $\alpha_*$  - dominated set-valued mappings fulfilling new generalised Ciric type rational contraction on a sequence contained in a closed ball in a complete dislocated  $b$ -metric space. As an application, we established the existence of common fixed point of multi  $\leq$ - dominated mappings in ordered complete dislocated  $b$ -metric space. Fixed point results with graphic contractions on a closed ball for graph dominated mappings were established. New definition and results for single-valued mappings were also given. Examples were given to demonstrate the variety of obtained results. The obtained results extended and generalised several comparable results in the existing literature.

**Keywords:**  $\alpha_*$  - dominated set-valued mapping, closed ball, common fixed point, complete dislocated  $b$ -metric space, graph dominated mapping, partial order.

## INTRODUCTION

Let  $M : W \rightarrow P(W)$  be a multifunction. A point  $w \in W$  is said to be a fixed point of  $M$  if  $w \in Mw$ . Fixed point theory plays a fundamental role in functional analysis and it has a large number of applications (Leibovic, 1964; Medrano-Cerda, 1987; Amar *et al.*, 2008; Jeribi *et al.*, 2013). Construction of fixed point results in ordered spaces for dominated mappings, fulfilling certain conditions on a subset of domains instead of a whole space, have been the recent research trend (Arshad *et al.*, 2013a, b, c; 2014; 2017; Hussain *et al.*, 2014; Beg *et al.*, 2015).

Hussain *et al.* (2013) introduced the concept of dislocated  $b$ -metric space as a generalisation of dislocated metric space (Hitzler & Seda, 2000) and  $b$ -metric space (Aydi *et al.*, 2012; Hussain *et al.*, 2013; Chifu & Petrusel, 2014; Bota *et al.*, 2016; Sintunavarat, 2016; Yamaod *et al.*, 2016; Mongkolkeha *et al.*, 2017). They established some common fixed point results in partially ordered dislocated  $b$ -metric space.

Nadler (1969) started the investigation of fixed point results for the multivalued functions. Asl *et al.* (2012) gave the idea of  $\alpha_*$ -admissible set-valued mapping and generalised some fixed point results for closed-valued multifunction in ordered metric space. For further results of  $\alpha_*$ -admissible set-valued mappings refer Ali *et al.* (2015) and Sintunavarat (2016). Recently, Alofi *et al.* (2017) introduced the concept of  $\alpha_*$ -dominated multivalued mappings and established some fixed point results for multi  $\leq$ -dominated mappings in an ordered complete dislocated quasi  $b$ -metric space. For further results of  $\alpha$ -dominated multivalued mappings refer Rasham *et al.* (2018a, b). This paper establish a common fixed point of  $\alpha_*$ -dominated set-valued mappings satisfying a generalised contraction in complete dislocated  $b$ -metric spaces. As an application, the existence of common fixed point of multi  $\leq$ -dominated mappings in an ordered complete dislocated  $b$ -metric space is established. The notion of multi-graph dominated mapping is introduced and the results with graphic contractions on a closed ball for such mappings are established. New definition

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and results for single-valued mappings are also given. compelling results in metric space, partial metric space and dislocated metric space can be obtained as applications of theorems, which are still not available in literature.

**METHODOLOGY**

Necessary and sufficient constraints are required to find the unique solution of a problem. These constraints are called contractive conditions. In most of the fixed point results, the contractive condition holds partially but the problem has a solution. In many cases contractive conditions do not satisfy the entire space  $X$  but the mappings have a common fixed point. To find out solution of such problems, we established an iterative sequence  $\{TS(x_n)\}$  based on two multivalued mappings  $S, T : X \rightarrow P(X)$ . The following definitions are necessary to understand the method.

**Definition 1.1.** (Hussain et al., 2013) Let  $H$  be a nonempty set and let  $d_b : H \times H \rightarrow [0, \infty)$  be a function called a dislocated  $b$ -metric (or simply  $d_b$ -metric), if for any  $x, y, z \in H$ ; and  $b \geq 1$  the following conditions hold:

- (i) If  $d_b(z, y) = 0$ , then  $z = y$ ;
- (ii)  $d_b(z, y) = d_b(y, z)$ ;
- (iii)  $d_b(z, y) \leq b[d_b(z, x) + d_b(x, y)]$ .

The pair  $(H, d_b)$  is called a dislocated  $b$ -metric space. It should be noted that the class of  $(H, d_b)$  metric spaces is effectively larger than that of dislocated metric spaces  $(H, d_1)$ , since every  $d_1$  metric is a  $d_b$  metric when  $b = 1$ .

It is clear that if  $d_b(z, y) = 0$ , then from (i),  $z = y$ . However, if  $z = y$ ,  $d_b(z, y)$  may not be 0. For  $z \in H$  and  $\varepsilon > 0$ ,  $\overline{B}(z, \varepsilon) = \{y \in H : d_b(z, y) \leq \varepsilon\}$  is a closed ball in  $(H, d_b)$ .

**Example 1.2.** (Hussain et al., 2013) If  $H = \mathbb{R}^+ \cup \{0\}$ , then  $d_b(z, y) = (z + y)^2$  defines a dislocated  $b$ -metric  $d_b$  on  $H$  with  $b = 2$ .

**Definition 1.3.** (Hussain et al., 2013) Let  $(H, d_b)$  be a dislocated  $b$ -metric space.

- (i) A sequence  $\{z_s\}$  in  $(H, d_b)$  is called Cauchy sequence if given  $\varepsilon > 0$ , there corresponds  $s_0 \in \mathbb{N}$  such that for all  $s, r \geq s_0$  we have  $d_b(z_r, z_s) < \varepsilon$  or  $\lim_{s, r \rightarrow \infty} d_b(z_s, z_r) = 0$ .
- (ii) A sequence  $\{z_s\}$  is called a dislocated  $b$ -converges (for short  $d_b$ -converges) to  $z$  if  $\lim_{s \rightarrow \infty} d_b(z_s, z) = 0$ .

In this case  $z$  is called a  $d_b$ -limit of  $\{z_s\}$ .

- (iii)  $(H, d_b)$  is complete if every Cauchy sequence in  $H$  converges to a point  $z \in H$ .

**Definition 1.4.** Let  $K$  be a nonempty subset of dislocated  $b$ -metric space  $(H, d_b)$  and let  $z \in H$ . An element  $y_0 \in K$  is called a best approximation in  $K$  if,

$$d_b(z, K) = d_b(z, y_0), \text{ where } d_b(z, K) = \inf_{y \in K} d_b(z, y).$$

If each  $z \in H$  has at least one best approximation in  $K$ , then  $K$  is called a proximal set.

We denote  $P(H)$  to be the set of all closed proximal subsets of  $z$ : Let  $\Omega_b$ , where  $b \geq 1$ , denote the family of all nondecreasing functions  $\varpi_b : [0, +\infty) \rightarrow [0, +\infty)$  such that  $\sum_{k=1}^{+\infty} b^k \varpi_b^k(t) < +\infty$  and  $b\varpi_b(t) < t$  for all  $t > 0$ , where  $\varpi_b^k$  is the  $k^{th}$  iterate of  $\varpi_b$ . Further,  $b^{s+1} \varpi_b^{s+1}(t) = b^s b \varpi_b(\varpi_b^s(t)) < b^s \varpi_b^s(t)$ .

**Definition 1.5.** (Alofi et al., 2017) Let  $(H, d_b)$  be a dislocated  $b$ -metric space,  $S : H \rightarrow P(H)$  be a multivalued mapping and  $\alpha : H \times H \rightarrow [0, +\infty)$ . Let  $A \subseteq H$ , thereby  $S$  is  $\alpha_*$ -dominated on  $A$ , whenever  $\alpha_*(z, Sz) \geq 1$  for all  $z \in A$ , where  $\alpha_*(z, Sz) = \inf\{\alpha(z, b) : b \in Sz\}$ . If  $A = H$ , then  $S$  is  $\alpha_*$ -dominated. It is shown (Rasham et al., 2018) that there are mappings which are  $\alpha_*$ -dominated but not  $\alpha_*$ -admissible (Asl et al., 2012). We denote the set  $\{(z, y) \in H \times H : \alpha(z, y) \geq 1 \text{ or } \alpha(y, z) \geq 1\}$  by  $\nabla_\alpha$ .

**Definition 1.6.** The function  $H_{d_b} : P(H) \times P(H) \rightarrow \mathbb{R}^+$ , defined by

$$H_{d_b}(A, B) = \max\{\sup_{a \in A} d_b(a, B), \sup_{b \in B} d_b(A, b)\}$$

is called a dislocated Hausdorff  $b$ -metric on  $P(H)$ .

**Lemma 1.7.** Let  $(H, d_b)$  be a dislocated metric space. Let  $(P(H); H_{d_b})$  be a dislocated Hausdorff  $b$ -metric space on  $P(H)$ : Then, for all  $A, B \in P(H)$  and for each  $a \in A$  there exists  $b_a \in B$  satisfies  $d_b(a, B) = d_b(a, b_a)$  then  $H_{d_b}(A, B) \geq d_b(a, b_a)$ .

**Construction of sequence.** Let  $(H, d_b)$  be a dislocated  $b$ -metric space,  $z_0 \in H$  and  $S, T : H \rightarrow P(H)$  be the multifunctions on  $H$ . Let  $z_1 \in Sz_0$  be an element such that  $d_b(z_0, Sz_0) = d_b(z_0, z_1)$ ; Let  $z_2 \in Tz_1$  be such that  $d_b(z_1, Tz_1) = d_b(z_1, z_2)$ ; Let  $z_3 \in Sz_2$  be such that  $d_b(z_2, Sz_2) = d_b(z_2, z_3)$ . Continuing this process, we construct a sequence  $z_s$  of points in  $H$  such that  $z_{2s+1} \in Sz_{2s}$  and  $z_{2s+2} \in Tz_{2s+1}$ , where  $s = 0, 1, 2, \dots$ . Further  $d_b(z_{2s}, Sz_{2s}) = d_b(z_{2s}, z_{2s+1})$ ,  $d_b(z_{2s+1}, Tz_{2s+1}) = d_b(z_{2s+1}, z_{2s+2})$ . We denote this iterative sequence by  $\{TS(z_s)\}$  and that  $\{TS(z_s)\}$  is a sequence in  $H$  generated by  $z_0$ .

For  $z, y \in H, a > 0$ , we define  $D_b(z, y)$  as

$$D_b(z, y) = \max\{d_b(z, y), \frac{d_b(z, Sz) \cdot d_b(y, Ty)}{a + d_b(z, y)}, d_b(z, Sz), d_b(y, Ty)\}.$$

## RESULTS AND DISCUSSION

### Results for $\alpha_*$ -dominated and $\leq$ -dominated multivalued mappings

**Theorem 2.1.** Let  $(H, d_b)$  be a complete dislocated  $b$ -metric space. Suppose there exist a function  $\alpha : H \times H \rightarrow [0, \infty)$ . Let,  $r > 0, z_0 \in \overline{B_{d_b}(z_0, r)}, S, T : H \rightarrow P(H)$  be two  $\alpha_*$ -dominated mappings on  $B_{d_b}(z_0, r)$  and  $\{TS(z_s)\}$  be a sequence in  $H$  generated by  $z_0$ . Assume that, for some  $\varpi_b \in \Omega_b$ , the following holds:

$$H_{d_b}(Sz, Ty) \leq \varpi_b(D_b(z, y)), \quad \dots(1)$$

whenever  $z, y \in \overline{B_{d_b}(z_0, r)} \cap \{TS(z_s)\}$  and  $(z, y) \in \nabla_\alpha$ .

Further,  $\sum_{k=0}^s b^{k+1} \{\varpi_b^k(d_b(z_0, Sz_0))\} \leq r$  for all  $s \in \mathbb{N} \cup \{0\}$  and  $b \geq 1$ .

$$\dots(2)$$

Then  $\{TS(z_s)\}$  is a sequence in  $\overline{B_{d_b}(z_0, r)}$ ,  $\alpha(z_s, z_{s+1}) \geq 1$  for all  $s \in \mathbb{N} \cup \{0\}$  and  $\{TS(z_s)\} \rightarrow z^* \in \overline{B_{d_b}(z_0, r)}$ . Also if the inequality (1) holds for  $z^*$  and  $(z_s, z^*) \in \nabla_\alpha$  for all  $s \in \mathbb{N} \cup \{0\}$ , then  $S$  and  $T$  have common fixed point  $z^*$  in  $B_{d_b}(z_0, r)$  and  $d_b(z^*, z^*) = 0$ .

**Proof.** Consider a sequence  $\{TS(z_s)\}$ . From equation (2), we get

$$d_b(z_0, z_1) \leq \sum_{k=0}^s b^{k+1} \{\varpi_b^k(d_b(z_0, Sz_0))\} \leq r.$$

It follows that,  $z_1 \in \overline{B_{d_b}(z_0, r)}$ .

Let  $z_2, \dots, z_l \in \overline{B_{d_b}(z_0, r)}$  for some  $l \in \mathbb{N}$ . If  $l = 2k+1$ , where  $k = 1, 2, \dots, \frac{l-1}{2}$ .

Since  $S, T : H \rightarrow P(H)$  be a  $\alpha_*$ -dominated mapping on  $B_{d_b}(z_0, r)$ , so  $\alpha_*(z_{2k}, Sz_{2k}) \geq 1$  and  $\alpha_*(z_{2k}, Sz_{2k}) \geq 1$ . As  $\alpha_*(z_{2k}, Sz_{2k}) \geq 1$ , this implies  $\inf\{\alpha(z_{2k}, b) : b \in Sz_{2k}\} \geq 1$ . Further,  $z_{2k+1} \in Sz_{2k}$ , therefore  $\alpha(z_{2k}, z_{2k+1}) \geq 1$ . Now by using Lemma 1.7, we obtain,

$$\begin{aligned} d_b(z_{2k+1}, z_{2k+2}) &\leq H_{d_b}(Sz_{2k}, Tz_{2k+1}) \leq \varpi_b(D_b(z_{2k}, z_{2k+1})) \\ &\leq \varpi_b(\max\{d_b(z_{2k}, z_{2k+1}), \\ &\frac{d_b(z_{2k}, z_{2k+1}) \cdot d_b(z_{2k+1}, z_{2k+2})}{a + d_b(z_{2k}, z_{2k+1})}, \\ &d_b(z_{2k}, z_{2k+1}), d_b(z_{2k+1}, z_{2k+2})\}) \end{aligned}$$

$$\leq \varpi_b(\max\{d_b(z_{2k}, z_{2k+1}), d_b(z_{2k+1}, z_{2k+2})\}).$$

If  $\max\{d_b(z_{2k}, z_{2k+1}), d_b(z_{2k+1}, z_{2k+2})\} = d_b(z_{2k+1}, z_{2k+2})$ , then

$$\begin{aligned} d_b(z_{2k+1}, z_{2k+2}) &\leq \varpi_b(d_b(z_{2k+1}, z_{2k+2})) \\ &\leq b\varpi_b(d_b(z_{2k+1}, z_{2k+2})). \end{aligned}$$

This is the contradiction to the fact that  $b\varpi_b(t) < t$  for all  $t > 0$ . Therefore,  $\max\{d_b(z_{2k}, z_{2k+1}), d_b(z_{2k+1}, z_{2k+2})\} = d_b(z_{2k}, z_{2k+1})$ .

Hence, we obtain,

$$d_b(z_{2k+1}, z_{2k+2}) \leq \varpi_b(d_b(z_{2k}, z_{2k+1})). \quad \dots(3)$$

As  $\alpha_*(z_{2k-1}, Tz_{2k-1}) \geq 1$  and  $z_{2k} \in Tz_{2k-1}$ , then  $\alpha(z_{2k-1}, z_{2k}) \geq 1$ . Now, by using Lemma 1.7, we have

$$\begin{aligned} d_b(z_{2k}, z_{2k+1}) &\leq H_{d_b}(Tz_{2k-1}, Sz_{2k}) \leq \varpi_b(D_b(z_{2k}, z_{2k-1})) \\ &\leq \varpi_b(\max\{d_b(z_{2k}, z_{2k-1}), \\ &\frac{d_b(z_{2k}, z_{2k+1}) \cdot d_b(z_{2k-1}, z_{2k})}{a + d_b(z_{2k}, z_{2k-1})}, \\ &d_b(z_{2k}, z_{2k+1}), d_b(z_{2k-1}, z_{2k})\}) \\ &\leq \varpi_b(\max\{d_b(z_{2k}, z_{2k-1}), d_b(z_{2k}, z_{2k+1})\}). \end{aligned}$$

If  $\max\{d_b(z_{2k}, z_{2k-1}), d_b(z_{2k}, z_{2k+1})\} = d_b(z_{2k}, z_{2k+1})$  then

$$d_b(z_{2k}, z_{2k+1}) \leq \varpi_b(d_b(z_{2k}, z_{2k+1})) \leq b\varpi_b(d_b(z_{2k}, z_{2k+1})).$$

This is a contradiction to the fact that  $b\varpi_b(t) < t$  for all  $t > 0$ . Hence, we obtain

$$d_b(z_{2k}, z_{2k+1}) \leq \varpi_b(d_b(z_{2k-1}, z_{2k})). \quad \dots(4)$$

As  $\varpi_b$  is nondecreasing,

$$\varpi_b(d_b(z_{2k}, z_{2k+1})) \leq \varpi_b(\varpi_b(d_b(z_{2k-1}, z_{2k}))).$$

By using the above inequality in equation (3), we have

$$d_b(z_{2k+1}, z_{2k+2}) \leq \varpi_b^2(d_b(z_{2k-1}, z_{2k})).$$

Continuing in this way, we obtain

$$d_b(z_{2k+1}, z_{2k+2}) \leq \varpi_b^{2k+1}(d_b(z_0, z_1)). \quad \dots(5)$$

Now, if  $l = 2k$ , where  $k = 1, 2, \dots, \frac{l}{2}$ , by using equation (4) and similar procedure as above, we have,

$$d_b(z_{2k}, z_{2k+1}) \leq \varpi_b^{2k}(d_b(z_0, z_1)). \quad \dots(6)$$

Now, by combining equations (5) and (6),

$$d_b(z_l, z_{l+1}) \leq \varpi_b^l(d_b(z_0, z_1)) \text{ for all } l \in \mathbb{N} \quad \dots(7)$$

Now, by using triangle inequality and by equation (7), we have

$$\begin{aligned} d_b(z_0, z_{l+1}) &\leq bd_b(z_0, z_1) + b^2d_b(z_1, z_2) + \dots + b^{l+1}d_b(z_l, z_{l+1}) \\ &\leq bd_b(z_0, z_1) + b^2\varpi_b(d_b(z_0, z_1)) + \dots + b^{l+1}\varpi_b^l(d_b(z_0, z_1)) \\ &\leq \sum_{k=0}^l b^{k+1}\{\varpi_b^k(d_b(z_0, z_1))\} \leq r. \end{aligned}$$

Thus  $z_{l+1} \in \overline{B_{d_b}(z_0, r)}$ . Hence  $z_s \in \overline{B_{d_b}(z_0, r)}$  for all  $s \in \mathbb{N}$ , and therefore  $\{TS(z_s)\}$  is a sequence in  $\overline{B_{d_b}(z_0, r)}$ . As  $S, T$  are  $\alpha_*$ -dominated on  $\overline{B_{d_b}(z_0, r)}$ ,  $\alpha_*(z_{2s}, Sz_{2s}) \geq 1$  and  $\alpha_*(z_{2s+1}, Tz_{2s+1}) \geq 1$ . This implies  $\alpha(z_s, z_{s+1}) \geq 1$  for all  $s \in \mathbb{N}$ . Also inequality in equation (7) can be written as

$$d_b(z_s, z_{s+1}) \leq \varpi_b^s(d_b(z_0, z_1)), \text{ for all } s \in \mathbb{N}. \quad \dots(8)$$

As  $\sum_{k=1}^{+\infty} b^k \varpi_b^k(t) < +\infty$ , then the series  $\sum_{k=1}^{+\infty} b^k \varpi_b^k(\varpi_b^{p-1}(d_b(z_0, z_1)))$  converges for each  $p \in \mathbb{N}$ .

As  $b\varpi_b(t) < t$ , so

$$b^{s+1}\varpi_b^{s+1}(\varpi_b^{p-1}(d_b(z_0, z_1))) < b^s\varpi_b^s(\varpi_b^{p-1}(d_b(z_0, z_1))) \text{ for all } s \in \mathbb{N}.$$

Fix  $\varepsilon > 0$ , then there exists  $p(\varepsilon) \in \mathbb{N}$ , such that

$$b\varpi_b(\varpi_b^{p(\varepsilon)-1}(d_b(z_0, z_1))) + b^2\varpi_b^2(\varpi_b^{p(\varepsilon)-1}(d_b(z_0, z_1))) + \dots < \varepsilon$$

Let  $s, r \in \mathbb{N}$  with  $r > s > p(\varepsilon)$ , then, we obtain

$$\begin{aligned} d_b(z_s, z_r) &\leq bd_b(z_s, z_{s+1}) + b^2d_b(z_{s+1}, z_{s+2}) + \dots + b^{r-s}d_b(z_{r-1}, z_r) \\ &\leq b\varpi_b^s(d_b(z_0, z_1)) + b^2\varpi_b^{s+1}(d_b(z_0, z_1)) + \dots \\ &\quad + b^{r-s}\varpi_b^{r-1}(d_b(z_0, z_1)) \\ &= b\varpi_b(\varpi_b^{s-1}(d_b(z_0, z_1))) + \dots + b^{r-s}\varpi_b^{r-s}(\varpi_b^{s-1}(d_b(z_0, z_1))) \\ &< b\varpi_b(\varpi_b^{p(\varepsilon)-1}(d_b(z_0, z_1))) + b^2\varpi_b^2(\varpi_b^{p(\varepsilon)-1}(d_b(z_0, z_1))) + \dots < \varepsilon. \end{aligned}$$

Thus we proved that  $\{TS(z_s)\}$  is a Cauchy sequence in  $(\overline{B_{d_b}(z_0, r)}, d_b)$ . As every closed ball in a complete dislocated  $b$ -metric space is complete, there exists,

$$z^* \in \overline{B_{d_b}(z_0, r)} \text{ such that } \{TS(z_s)\} \rightarrow z^* \text{ that is}$$

$$\lim_{s \rightarrow \infty} d_b(z_s, z^*) = 0 \quad \dots(9)$$

Now,

$$\begin{aligned} d_b(z^*, Sz^*) &\leq bd_b(z^*, z_{2s+2}) + bd_b(z_{2s+2}, Sz^*) \\ &\leq bd_b(z^*, z_{2s+2}) + bH_{d_b}(Tz_{2s+1}, Sz^*). \end{aligned} \quad \text{(by Lemma 1.7)}$$

Since  $\alpha(z^*, z_{2s+1}) \geq 1$ , we obtain

$$\begin{aligned} d_b(z^*, Sz^*) &\leq bd_b(z^*, z_{2s+2}) + b\varpi_b(\max\{d_b(z^*, z_{2s+1}), d_b(z^*, Sz^*), \\ &\quad \frac{d_b(z^*, Sz^*) \cdot d_b(z_{2s+1}, z_{2s+2})}{a + d_b(z^*, z_{2s+1})}, d_b(z_{2s+1}, z_{2s+2})\}). \end{aligned}$$

Letting  $s \rightarrow \infty$ , and using the inequality in equation (9), we obtain  $d_b(z^*, Sz^*) \leq b\varpi_b(d_b(z^*, Sz^*))$ . A contradiction, hence  $d_b(z^*, Sz^*) = 0$  or  $z^* \in Sz^*$ . Similarly, by using,

$$d_b(z^*, Tz^*) \leq bd_b(z^*, z_{2s+1}) + bd_b(z_{2s+1}, Tz^*)$$

we can show that  $z^* \in Tz^*$ . Hence,  $S$  and  $T$  have a common fixed point  $z^*$  in  $\overline{B_{d_b}(z_0, r)}$ . Now,

$$d_b(z^*, z^*) \leq b[d_b(z^*, Tz^*) + d_b(Tz^*, z^*)] \leq 0 \quad \blacksquare$$

This implies that,  $d_b(z^*, z^*) = 0$ .

We have the following result without closed ball and  $\alpha$ -dominated mappings for one multivalued mapping.

**Theorem 2.2.** Let  $(H, d_b)$  be a complete dislocated  $b$ -metric space. Suppose  $S : H \rightarrow P(H)$  be a multivalued mapping: Assume that, for some  $\varpi_b \in \Omega_b$ , the following hold:

$$H_{d_b}(Sz, Sy) \leq \varpi_b(D_b(z, y))$$

for all  $z, y \in \{SS(z_s)\}$ . Then  $\{SS(z_s)\} \rightarrow z^* \in H$  and  $S$  has a fixed point  $z^*$  in  $H$ : Also  $d_b(z^*, z^*) = 0$ .

**Definition 2.3.** (Alofi et al., 2017) Let  $H$  be a nonempty set,  $\preceq$  is a partial order on  $H$ ,  $a \in H$  and  $B \subseteq H$ . We say that  $a \preceq B$  whenever  $a \preceq b$  for all  $b \in B$ . A mapping  $S : H \rightarrow P(H)$  is said to be multi  $\preceq$ -dominated on  $A$  if  $a \preceq Sa$  for each  $a \in A \subseteq H$ . If  $A = H$ , then  $S : H \rightarrow P(H)$  is said to be multi  $\preceq$ -dominated. We denote the set  $\{(z, y) \in H \times H : z \preceq y \text{ or } y \preceq z\}$  by  $\nabla_{\preceq}$ .

We have the following result for multi  $\preceq$ -dominated mappings on  $\overline{B_{d_b}(z_0, r)}$  in an ordered complete dislocated  $b$ -metric space.

**Theorem 2.4.** Let  $(H, \preceq, d_b)$  be an ordered complete

dislocated  $b$ -metric space. Let,  $r > 0, z_0 \in B_{d_b}(z_0, r), S, T : H \rightarrow P(H)$  be two multi  $\preceq$ -dominated mappings on  $\overline{B_{d_b}(z_0, r)}$  and  $\{TS(z_s)\}$  be a sequence in  $H$  generated by  $z_0$ . Assume that, for some  $\varpi_b \in \Omega_b$ , the following hold.

$$H_{d_b}(Sz, Ty) \leq \varpi_b(D_b(z, y)), \quad \dots(10)$$

whenever  $z, y \in \overline{B_{d_b}(z_0, r)} \cap \{TS(z_s)\}$  and  $(z, y) \in \nabla_{\preceq}$ .

Also

$$\sum_{k=0}^s b^{k+1} \{\varpi_b^k(d_b(z_0, z_1))\} \leq r \text{ for all } s \in \mathbb{N} \cup \{0\} \text{ and } b \geq 1. \quad \dots(11)$$

Then  $\{TS(z_s)\}$  is a sequence in  $\overline{B_{d_b}(z_0, r)}$ ,  $z_s \preceq z_{s+1}$  and  $\{TS(z_s)\} \rightarrow z^* \in \overline{B_{d_b}(z_0, r)}$ . Also if the inequality equation (10) holds for  $z^*$  and  $(z_s, z^*) \in \nabla_{\preceq}$  for all  $s \in \mathbb{N} \cup \{0\}$ . Then  $S$  and  $T$  have a common fixed point  $z^*$  in  $B_{d_b}(z_0, r)$  and  $d_b(z^*, z^*) = 0$ .

**Proof.** Let  $\alpha : H \times H \rightarrow [0, +\infty)$  be a mapping defined by  $\alpha(z, y) = 1$  for all  $z \in \overline{B_{d_b}(z_0, r)}$  and  $(z, y) \in \nabla_{\preceq}$ , and  $\alpha(z, y) = 0$  for all other elements  $z, y \in H$ . As  $S$  and  $T$  are the dominated mappings on  $\overline{B_{d_b}(z_0, r)}$ , so  $z \preceq Sz$  and  $z \preceq Tz$  for all  $z \in \overline{B_{d_b}(z_0, r)}$ . This implies that  $z \preceq b$  for all  $b \in Sz$  and  $z \preceq c$  for all  $c \in Tz$ . So,  $\alpha(z, b) = 1$  for all  $b \in Sz$  and  $\alpha(z, c) = 1$  for all  $c \in Tz$ .

This implies that  $\inf\{\alpha(z, y) : y \in Sz\} = 1$  and  $\inf\{\alpha(z, y) : y \in Tz\} = 1$ . Hence  $\alpha_*(z, Sz) = 1, \alpha_*(z, Tz) = 1$  for all  $z \in B_{d_b}(z_0, r)$ . So,  $S, T : H \rightarrow P(H)$  are the  $\alpha_*$ -dominated mapping on  $\overline{B_{d_b}(z_0, r)}$ . Moreover, inequality in equation (10) can be written as,

$$H_{d_b}(Sz, Ty) \leq \varpi_b(D_b(z, y)),$$

whenever  $z, y \in \overline{B_{d_b}(z_0, r)} \cap \{TS(z_s)\}$  and  $(z, y) \in \nabla_{\alpha}$ . Also, inequality in equation (11) holds. Then, by Theorem 2.1, we have  $\{TS(z_s)\}$  is a sequence in  $B_{d_b}(z_0, r)$ ,  $z_s \preceq z_{s+1}$  and  $\{TS(z_s)\} \rightarrow z^* \in \overline{B_{d_b}(z_0, r)}$ . Now,  $z_s, z^* \in \overline{B_{d_b}(z_0, r)}$  and  $(z_s, z^*) \in \nabla_{\preceq}$  implies that  $(z_s, z^*) \in \nabla_{\alpha}$  for all  $s \in \mathbb{N} \cup \{0\}$ . Therefore all the conditions of Theorem 2.1 are satisfied. Hence, by Theorem 2.1,  $S$  and  $T$  have a common fixed point  $z^*$  in  $\overline{B_{d_b}(z_0, r)}$  and  $d_b(z^*, z^*) = 0$ .

We have the following result without closed ball in an ordered complete dislocated  $b$ -metric space. The result is only for one multivalued mapping.

**Theorem 2.5.** Let  $(H, \preceq, d_b)$  be an ordered complete dislocated  $b$ -metric space. Let  $S : H \rightarrow P(H)$  be a multi  $\preceq$ -dominated mapping on  $H$ . Assume that, for some  $\varpi_b \in \Omega_b$ , the following holds:

$$H_{d_b}(Sz, Sy) \leq \varpi_b(D_b(z, y)) \quad \dots(12)$$

whenever  $z, y \in \{SS(z_s)\}$  and  $(z, y) \in \nabla_{\preceq}$ . Then,  $\{SS(z_s)\} \rightarrow z^* \in H$  and  $z_s \preceq z_{s+1}$ . Further, if the inequality in equation (12) holds for  $z^*$  and  $(z_s, z^*) \in \nabla_{\preceq}$  for all  $s \in \mathbb{N} \cup \{0\}$ . Then  $S$  has a fixed point  $z^*$  and  $d_b(z^*, z^*) = 0$ .

**Example 2.6.** Let  $H = \mathbb{Q}^+ \cup \{0\}$  and let  $d_b : H \times H \rightarrow [0, \infty)$  be the complete dislocated  $b$ -metric on  $H$  defined by

$$d_b(z, y) = (z + y)^2 \text{ for all } z, y \in H$$

with parameter  $b = 2$ . Define the multivalued mappings,  $S, T : H \rightarrow P(H)$  by,

$$Sa = \begin{cases} \left[ \frac{a}{3}, \frac{2}{3}a \right] \cap H & \text{if } a \in [0, 19] \cap H \\ [a, a + 1] \cap H & \text{if } a \in (19, \infty) \cap H \end{cases}$$

and

$$Tb = \begin{cases} \left[ \frac{b}{4}, \frac{3}{4}b \right] \cap H & \text{if } b \in [0, 19] \cap H \\ [b + 1, b + 3] \cap H & \text{if } b \in (19, \infty) \cap H. \end{cases}$$

Considering,  $z_0 = 1, r = 400$ , then  $\overline{B_{d_b}(z_0, r)} = [0, 19] \cap H$ . Now  $d_b(z_0, Sz_0) = d_b(1, S1) = d_b(1, \frac{1}{3}) = \frac{16}{9}$ . Therefore we obtain a sequence  $\{TS(z_s)\} = \{1, \frac{1}{3}, \frac{1}{12}, \frac{1}{36}, \dots\}$  in  $H$  generated by  $z_0$ . Let  $\varpi_b(t) = \frac{4t}{10}$ , then  $b\varpi_b(t) < t$ .

Define

$$\alpha(z, y) = \begin{cases} 1 & \text{if } z > y \\ \frac{1}{2} & \text{otherwise} \end{cases}.$$

Then  $S, T : H \rightarrow P(H)$  be the  $\alpha_*$ -dominated mappings on  $\overline{B_{d_b}(z_0, r)}$ . Now take  $20, 21 \in H$  and  $a = 1$ , then, we have,

$$H_{d_b}(S20, T21) = 1936 > \varpi_b(D_l(z, y)) = \frac{7396}{10}.$$

Therefore, the contractive condition does not hold the whole space  $H$ . Now for all  $z, y \in \overline{B_{d_b}(z_0, r)} \cap \{TS(z_s)\}$  with  $(z, y) \in \nabla_{\alpha}$ , we have

$$\begin{aligned} H_{d_b}(Sz, Ty) &= \max\{d_b(\frac{2z}{3}, \frac{y}{4}), d_b(\frac{z}{3}, \frac{3y}{4})\} \\ &= \max\left\{\left(\frac{2z}{3} + \frac{y}{4}\right)^2, \left(\frac{z}{3} + \frac{3y}{4}\right)^2\right\} \\ &\leq \varpi_b(\max\{(z + y)^2, \frac{25z^2y^2}{9(1 + (z + y)^2)}, \left(\frac{4z}{3}\right)^2, \left(\frac{5y}{4}\right)^2\}). \end{aligned}$$

Therefore, the contractive condition holds on  $\overline{B_{d_b}(z_0, r)} \cap \{TS(z_s)\}$ . Further, for all  $s \in \mathbb{N} \cup \{0\}$ , we have

$$\sum_{k=0}^s b^{k+1} \{\varpi_b^k(d_b(z_0, z_1))\} = \frac{16}{9} \times 2 \sum_{k=0}^s \left(\frac{4}{5}\right)^k < 400 = r.$$

Assume  $\{TS(z_s)\}$  is a sequence in  $\overline{B_{d_b}(z_0, r)}$ ,  $\alpha(z_s, z_{s+1}) \geq 1$  and  $\{TS(z_s)\} \rightarrow 0 \in \overline{B_{d_b}(z_0, r)}$ . Further,  $\alpha(z_s, 0) \geq 1$  for all  $s \in \mathbb{N} \cup \{0\}$ . Hence, all the conditions of Theorem 2.1 are satisfied. Moreover, 0 is a common fixed point of  $S$  and  $T$ .

**Results for graphic contractions**

Jachymski (2008) established fixed points for mappings satisfying graphic contraction. Let  $(H, d)$  be a metric space and  $\Delta$  represents the diagonal of the cartesian product  $H \times H$ . Assume that  $G$  is a directed graph. The set  $V(G)$  having vertices along with  $H$ , and the set  $E(G)$  denoted the edges of  $H$  included all loops, i.e.,  $E(G) \supseteq \Delta$ . If  $G$  has no parallel edges, then we can unify  $G$  with pair  $(V(G), E(G))$  (Bojor, 2012; Hussain et al., 2014; Tiammee & Suantai, 2014).

**Definition 3.1.** Let  $H$  be a nonempty set and  $G = (V(G), E(G))$  be a graph such that  $V(G) = H$ ,  $A \subseteq H$ . A mapping  $S : H \rightarrow P(H)$  is said to be multi graph dominated on  $A$  if  $(z, y) \in E(G)$ , for all  $y \in Sz$  and  $z \in A$ . We denote the set  $\{(z, y) \in H \times H : (z, y) \in E(G) \text{ or } (y, z) \in E(G)\}$  by  $\nabla_G$ .

**Theorem 3.2.** Let  $(H, d_b)$  be a complete dislocated  $b$ -metric space endowed with a graph  $G$ . Let,  $r > 0$ ,  $z_0 \in \overline{B_{d_b}(z_0, r)}$ ,  $S, T : H \rightarrow P(H)$  and  $\{TS(z_s)\}$  be a sequence in  $H$  generated by  $z_0$ . Assume that the following holds:

- (i)  $S$  and  $T$  are multi graph dominated on  $\overline{B_{d_b}(z_0, r)} \cap \{TS(z_s)\}$ ;
- (ii) there exists  $\varpi_b \in \Omega_b$ , such that

$$H_{d_b}(Sz, Ty) \leq \varpi_b(D_b(z, y)), \tag{13}$$

whenever  $z, y \in \overline{B_{d_b}(z_0, r)} \cap \{TS(z_s)\}$  and  $(z, y) \in \nabla_G$ ;

- (iii)  $\sum_{k=0}^s b^{k+1} \{\varpi_b^k(d_b(z_0, Sz_0))\} \leq r$  for all  $s \in \mathbb{N} \cup \{0\}$  and  $b \geq 1$ .

Then,  $\{TS(z_s)\}$  is a sequence in  $B_{d_b}(z_0, r)$ ,  $(z_s, z_{s+1}) \in E(G)$  and  $\{TS(z_s)\} \rightarrow z^*$ . Also, if the inequality in equation (13) holds for  $z^*$  and  $(z_s, z^*) \in \nabla_G$

for all  $s \in \mathbb{N} \cup \{0\}$ , then  $S$  and  $T$  have common fixed point  $z^*$  in  $\overline{B_{d_b}(z_0, r)}$  and  $d_b(z^*, z^*) = 0$ .

**Proof.** Define,  $\alpha : H \times H \rightarrow (0, \infty)$  by

$$\alpha(z, y) = \begin{cases} 1, & \text{if } z \in \overline{B_{d_b}(z_0, r)}, (z, y) \in \nabla_G \\ 0, & \text{otherwise.} \end{cases}$$

As  $S$  and  $T$  are graph dominated on  $\overline{B_{d_b}(z_0, r)}$ , then for  $z \in \overline{B_{d_b}(z_0, r)}$ ,  $(z, y) \in E(G)$  for all  $y \in Sz$  and  $(z, y) \in E(G)$  for all  $y \in Tz$ . So,  $\alpha(z, y) = 1$  for all  $y \in Sz$  and  $\alpha(z, y) = 1$  for all  $y \in Tz$ . This implies that  $\inf\{\alpha(z, y) : y \in Sz\} = 1$  and  $\inf\{\alpha(z, y) : y \in Tz\} = 1$ . Hence  $\alpha_*(z, Sz) = 1$ ,  $\alpha_*(z, Tz) = 1$  for all  $z \in \overline{B_{d_b}(z_0, r)}$ . Therefore  $S, T : H \rightarrow P(H)$  are the  $\alpha_*$ -dominated mappings on  $\overline{B_{d_b}(z_0, r)}$ . Moreover, inequality in equation (13) can be written as

$$H_{d_b}(Sz, Ty) \leq \varpi_b(D_b(z, y)),$$

whenever  $z, y \in \overline{B_{d_b}(z_0, r)} \cap \{TS(z_s)\}$  and  $(z, y) \in \nabla_G$ . Further, (iii) in Theorem 3.2 holds. Then, by Theorem 2.1,  $\{TS(z_s)\}$  is a sequence in  $\overline{B_{d_b}(z_0, r)}$  and  $\{TS(z_s)\} \rightarrow z^* \in \overline{B_{d_b}(z_0, r)}$ . Now,  $z_s, z^* \in \overline{B_{d_b}(z_0, r)}$  and  $(z_s, z^*) \in \nabla_G$  implies that  $(z_s, z^*) \in \nabla_G$ . Therefore, all the conditions of Theorem 2.1 are satisfied. Hence, by Theorem 2.1,  $S$  and  $T$  have a common fixed point  $z^*$  in  $B_{d_b}(z_0, r)$  and  $d_b(z^*, z^*) = 0$ .

We have the following result without closed ball in complete dislocated  $b$ -metric space for multi graph dominated mapping. Therefore we write the result only for one multivalued mapping and for  $D_b(z, y) = d_b(z, y)$ .

**Theorem 3.3.** Let  $(H, d_b)$  be a complete dislocated  $b$ -metric space endowed with a graph  $G$ . Let,  $r > 0$ ,  $z_0 \in \overline{B_{d_b}(z_0, r)}$ ,  $S : H \rightarrow P(H)$  and  $\{SS(z_s)\}$  be a sequence in  $H$  generated by  $z_0$ . Assume that the following holds:

- (i)  $S$  is a multi graph dominated on  $\{SS(z_s)\}$ ;
- (ii) there exists  $\varpi_b \in \Omega_b$  such that

$$H_{d_b}(Sz, Sy) \leq \varpi_b(d_b(z, y)), \tag{14}$$

whenever  $z, y \in \{TS(z_s)\}$  and  $(z, y) \in \nabla_G$ ;

Then,  $(z_s, z_{s+1}) \in E(G)$  and  $\{SS(z_s)\} \rightarrow z^*$ . Further, if the inequality in equation (14) holds for  $z^*$  and  $(z_s, z^*) \in \nabla_G$  for all  $s \in \mathbb{N} \cup \{0\}$ , then  $S$  and  $T$  have common fixed point  $z^*$  in  $H$  and  $d_b(z^*, z^*) = 0$ .

**Results for singlevalued mappings**

Recently, Arshad *et al.* (2017) have given the following definition for dislocated quasi metric space.

**Definition 4.1:** Let  $(H, d_b)$  be a dislocated  $b$ -metric space,  $T : H \rightarrow H$  be a self mapping,  $A \subseteq H$  and  $\alpha : H \times H \rightarrow [0, +\infty)$  be a function. We say that,

- (i)  $T$  is  $\alpha$ -dominated mapping on  $A$ , if  $\alpha(z, Tz) \geq 1$  for all  $z \in A$ .
- (ii)  $(H, d_b)$  is  $\alpha$ -regular on  $A$  if for any sequence  $\{z_s\}$  in  $A$  such that  $\alpha(z_s, z_{s+1}) \geq 1$  for all  $s \geq 0$  and  $z_s \rightarrow u \in A$  As  $s \rightarrow \infty$  we have  $\alpha(z_s, u) \geq 1$  for all  $s \geq 0$ .

**Theorem 4.2.** Let  $(H, d_b)$  be a complete dislocated  $b$ -metric space. Suppose there exists a function  $\alpha : H \times H \rightarrow [0, +\infty)$ . Let  $r > 0, z_0 \in B_{d_b}(z_0, r)$  and  $S, T : H \rightarrow H$  be two  $\alpha$ -dominated mappings on  $B_{d_b}(z_0, r)$ . Assume that, for some  $\varpi_b \in \Omega_b$ , the following holds:

$$d_b(Sz, Ty) \leq \varpi_b(D_b(z, y)),$$

whenever  $z, y \in \overline{B_{d_b}(z_0, r)}$  and  $(z, y) \in \nabla_\alpha$ . Further,

$$\sum_{k=0}^s b^{k+1} \{ \varpi_b^k(d_b(z_0, Sz_0)) \} \leq r \text{ for all } s \in \mathbb{N} \cup \{0\} \text{ and } b \geq 1.$$

If  $(H, d_b)$  is  $\alpha$ -regular on  $B_{d_b}(z_0, r)$ , then there exists a common fixed point  $z^*$  of  $S$  and  $T$  in  $\overline{B_{d_b}(z_0, r)}$  and  $d_b(z^*, z^*) = 0$ .

Recall that, (Arshad *et al.*, 2013) if  $(H, \preceq)$  be a partially ordered set. A self mapping  $f$  on  $H$  is called dominated if  $fz \preceq z$  for each  $z$  in  $H$ . Two elements  $z, y \in H$  are called comparable if  $z \preceq y$  or  $y \preceq z$  holds.

**Theorem 4.3.** Let  $(H, \preceq, d_b)$  be an ordered complete dislocated  $b$ -metric space,  $S, T : H \rightarrow H$  be dominated maps and  $z_0$  be an arbitrary point in  $H$ . Suppose that for some  $\varpi_b \in \Omega_b$  and for  $S \neq T$ , we have,

$$d_b(Sz, Ty) \leq \varpi_b(D_b(z, y)) \text{ for all comparable elements } z, y \text{ in } \overline{B(z_0, r)}. \text{ Further,}$$

$$\sum_{k=0}^s b^{k+1} \{ \varpi_b^k(d_b(z_0, Sz_0)) \} \leq r \text{ for all } s \in \mathbb{N} \cup \{0\} \text{ and } b \geq 1.$$

If for a nonincreasing sequence  $\{z_s\}$  in  $\overline{B(z_0, r)}$ ,  $\{z_s\} \rightarrow u$  implies that  $u \preceq z_s$  or  $z_s \preceq u$ . Then, there exists  $z^* \in \overline{B(z_0, r)}$  such that  $d_b(z^*, z^*) = 0$  and  $z^* = Sz^* = Tz^*$ .

By putting  $b = 1, D_l(z, y) = d_l(z, y)$  and  $\varpi(t) = kt$ , we obtain the main result. Theorem 2.1 in (Arshad *et al.* 2013) as a corollary of Theorem 4.3.

**Corollary 4.4.** (Arshad *et al.* 2013) Let  $(H, \preceq, d_l)$  be an ordered complete dislocated metric space,  $S, T : H \rightarrow H$  be dominated maps and  $z_0$  be an arbitrary point in  $H$ . Suppose that for  $k \in [0, 1)$  and for  $S \neq T$ , we have,

$$d_l(Sz, Ty) \leq kd_l(z, y) \text{ for all comparable elements } z, y \text{ in } \overline{B(z_0, r)} \text{ and } d_l(z_0, Sz_0) \leq (1 - k)r.$$

If for a non-increasing sequence  $\{z_s\}$  in  $\overline{B(z_0, r)}$ ,  $\{z_s\} \rightarrow u$  implies that  $u \preceq z_s$ . Then there exists  $z^* \in \overline{B(z_0, r)}$  such that  $d_l(z^*, z^*) = 0$  and  $z^* = Sz^* = Tz^*$ .

**Definition 4.5.** Let  $H$  be a nonempty set and  $G = (V(G), E(G))$  be a graph such that  $V(G) = H, A \subseteq H$ . A mapping  $S : H \rightarrow H$  is said to be graph dominated on  $A$  if  $(z, Sz) \in E(G)$ , for all  $z \in A$ .

**Theorem 4.6.** Let  $(H, d_b)$  be a complete dislocated  $b$ -metric space endowed with a graph  $G$ . Let,  $r > 0, z_0 \in \overline{B_{d_b}(z_0, r)}$  and  $S, T : H \rightarrow H$ . Assume that the following holds:

- (i)  $S$  and  $T$  are graph dominated on  $\overline{B_{d_b}(z_0, r)}$ ;
- (ii) there exists  $\varpi_b \in \Omega_b$ , such that

$$d_b(Sz, Ty) \leq \varpi_b(D_b(z, y)),$$

for all  $z, y \in \overline{B_{d_b}(z_0, r)}$  and  $(z, y) \in \nabla_G$ ;

- (iii)  $\sum_{k=0}^s b^{k+1} \{ \varpi_b^k(d_b(z_0, Sz_0)) \} \leq r$  for all  $s \in \mathbb{N} \cup \{0\}$  and  $b \geq 1$ .

Then, there exists a sequence  $\{z_s\}$  in  $B_{d_b}(z_0, r)$  such that  $(z_s, z_{s+1}) \in E(G)$  and  $\{z_s\} \rightarrow z^* \in \overline{B_{d_b}(z_0, r)}$ . Further, if  $(z_s, z^*) \in \nabla_G$  for all  $s \in \mathbb{N} \cup \{0\}$ , then  $S$  and  $T$  have common fixed point  $z^*$  in  $\overline{B_{d_b}(z_0, r)}$  and  $d_b(z^*, z^*) = 0$ .

Now, we present only one new result in metric space. Many other results can be derived as corollaries of our previous results.

**Theorem 4.7** Let  $(H, d)$  be a complete metric space endowed with a graph  $G$  and  $S : H \rightarrow H$  be a mapping. Assume that the following holds:

- (i)  $S$  is a graph dominated on  $H$ ;
- (ii) there exists  $k \in [0, 1)$  such that

$$d(Sz, Sy) \leq kd(z, y),$$

for all  $z, y \in H$  and  $(z, y) \in \nabla_G$ ;

Then, there exist a sequence  $\{z_s\}$  such that  $(z_s, z_{s+1}) \in E(G)$  and  $\{z_s\} \rightarrow z^*$ . Further, if  $(z_s, z^*) \in \nabla_G$  for all  $s \in \mathbb{N} \cup \{0\}$ , then  $S$  has fixed point  $z^*$  in  $H$ .

## CONCLUSION

The present study obtained fixed point results for new generalised contraction on an intersection of a closed ball and a sequence for a more general class of  $\alpha_*$ -dominated mappings rather than  $\alpha_*$ -admissible mappings. Results with graphic contractions on a closed ball for graph dominated mappings are established, and Theorems for ordered multi  $\preceq$ -dominated mappings are obtained. Moreover, we investigated the obtained results as a new framework of dislocated  $b$ -metric space. New results in partial  $b$ -metric space, dislocated metric space, partial metric space,  $b$ -metric space and metric space can be obtained as corollaries of the obtained results.

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