

UNFOLDING PARTICLE SIZE DISTRIBUTIONS OF TEA GRADES

1—HIGH GROWN BROKEN ORANGE PEKOE AND BROKEN ORANGE PEKOE FANNINGS

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Grades of tea are popularly described by various names, which imply that they consist of particles of certain sizes. High-grown Broken Orange Pekoe (BOP) and Broken Orange Pekoe Fannings (BOPF) grades are polydisperse mixtures of sizes, which have been found to be distributed lognormally. These grades have been completely characterized by the mean and standard deviations of the particle size distributions measured on a logarithmic scale. Two parameters which are related and which are more convenient to adopt, are the mean 'Fineness Modulus' of particles in a grade and their standard deviation. These parameters have been defined and tolerance limits of these for well-sorted BOP and BOPF grades have been determined by sieve separation followed by probit analysis. The procedure for using this criterion to judge whether any batch or sample conforms to acceptable size standards is described. This information can be used for process control in tea manufacture.

A knowledge of the particle size distribution of tea grades is of considerable importance in testing them for conformity to specifications and in exercising control over manufacturing processes. Grades such as Broken Orange Pekoe, Broken Orange Pekoe Fannings, Dust *etc.*, are polydisperse mixtures of particle sizes. Ideally, these grades should contain particles within very broad size ranges. Incorrect sorting techniques could, however, lead to grades having particles, which fall outside even these broad size ranges. These grades are described on the basis of visual observations as 'well sorted', 'uneven', 'too large in size' *etc.*, by the tea trade. The analysis presented attempts to define well-sorted, high-grown BOP and BOPF grades in terms of particle size distributions. This knowledge, it is hoped, will help to produce grades, which conform to existing size standards.

Adoption of standards in the particle size distributions of grades would help define the efficiency of sorting techniques. Roller and roll-breaker efficiency too could be defined and roll-breaking operations could be geared to minimize oversize particles in the dhools. The benefit that would accrue from this is that size reduction processes such as cutting and nipping of oversize particles of fired tea during grading, could be minimized.

Earlier attempts to specify standards for graded tea have not been so successful, although the need for such standards has been stressed (Evans 1933). Apart from reporting results obtained by fractionating grades into three broad nominal particle size ranges, in one series of experiments (Evans 1931; 1932) and into eight nominal particle size ranges in another series of experiments (Lamb 1937a; 1937b; 1939; 1940), no attempt has been made either to define grades on the basis of particle size distribution or to define the efficiency of sorting machines and techniques.

Sieve analysis

Of the many methods available, only sieve separation was adopted in the present series of experiments. This method was considered to be the best available for the study of broken tea grades because roll breaking and grading operations are solely dependent on the use of sieves. Sieve analysis, however, may be unsatisfactory for characterizing grades such as Orange Pekoe (OP) and some of the other low-country grades, where large differences are observed between the maximum and minimum dimensions of particles. In the case of OP, the average of the maximum dimensions of the particles is the important factor and sieve analysis would fail to give reliable estimates of it. For this reason, such grades will be left out in this study, and the results reported on refer to only small-leaf teas such as high-grown BOP and BOPF.

Mathematical model

Particle size distribution is the functional relation of the distribution with respect to the size of individual particles. The most complete description of a poly-disperse mixture of particles is given by its particle size distribution. These distributions can be plotted in terms of proportions present in each unit of diameter against the several diameters, which is essentially a differential method. Curves obtained in this manner for tea grades resemble probability curves, but are slightly skewed rather than symmetrical as in a normal probability curve. Alternatively cumulative per cent undersize could be plotted against the several diameters. This is an integral method, to give S-shaped curves resembling the ogive or integrated normal probability curve. For either of these methods it is common to employ a weight basis for percentage calculations (Work & Stern 1963).

Simple treatments of particle size distributions are possible, as in table form showing 10-50-90-% or the quadrants 25-50-75%. Both have value but are not so definitive as the full distribution. In some cases when the index of evenness of dhool is required, the significant value is the percentage of coarse particles larger than a specified size. In other cases, the per cent retained on some sieve helps to define coarseness. Merely stating that all, or all but a small percentage, passes a given sieve is inadequate to define the true fineness of a material. Complete particle size analysis to show the entire distribution or data which completely defines the distribution is essential for most comparisons and calculations.

Several equations have been proposed to correlate the quantity of particulate material with its particle size to obtain distribution relationships (Austin 1939). Of these the most successful is to plot particle size on a logarithmic scale and cumulative per cent undersize (or oversize) on a probability scale. Particle size distributions of high-grown broken tea grades can be represented by straight lines on logarithmic probability paper. The mathematical treatment of this relation to obtain properties of tea grades is developed below.

The almost perfect linear relationship between cumulative per cent undersize and particle size when plotted on logarithmic probability paper indicates that particle sizes in broken tea grades to be lognormally distributed. Mean logarithm of the size of particles and the standard deviation of the logarithm of the sizes would, therefore, completely characterize the distribution and the grade of tea. Two statistics which are related to these and defined in what follows, are the mean 'Fineness Modulus' (\bar{z}) and its standard deviation. These two latter statistics will be adopted for characterization of tea grades.

In a complete set of British Standard Test Sieves (B.S. 410, 1960), specifications of which are given in Appendix 2, the aperture size of one sieve is $4\sqrt{2}$ times as large as the next smaller sieve. Average size of particles transmitted through a sieve and retained on a sieve one step smaller, having a nominal No. N is defined by

$$D_N = \frac{1}{2} [d_N + 4\sqrt{2} d_N]$$

ie $D_N = \frac{1}{2} d_N [1 + 4\sqrt{2}]$ (1)

On the basis of the relationship defined by equation 1, the average size of particles retained on any sieve z steps larger than a sieve having a nominal No. N of a complete set of British Standard sieves is given by

$$D_{N+z} = ab^z$$
(2)

where $a = \frac{1}{2} d_N [1 + 4\sqrt{2}]$ (3)

$$b = 4\sqrt{2}$$
(4)

Broadly, following the classification system devised by D. A. Abrams (Henderson & Perry 1955), which is used by the American Society of Agricultural Engineers for determining the performance of feed grinders, the exponent of b in equation 2 is defined as the 'Fineness Modulus'.

$$F_{N+z} = z$$
(5)

A sieve having a nominal No. 60 is the smallest, that is likely to be required for particle size analysis of tea grades, and the 'Fineness Modulus' of particles retained on such a mesh is taken as zero.

$$F_{60} = 0$$
(6)

From equations 5 and 6, it follows that the Fineness Modulus of particles retained on Mesh No. 52 which is one step larger than No. 60 is 1. Proceeding in this manner, it is quite clear that the Fineness Moduli of particles retained on meshes from No. 52 to No. 5 are integral numbers from 1 to 15.

From equations 2 and 6 it is seen that

$$D_{60} = a = \frac{1}{2} d_{60} [1 + 4\sqrt{2}]$$

Since $d_{60} = 250$ microns it follows that

$$a = 125 [1 + 4\sqrt{2}] \quad \dots\dots\dots(7)$$

∴ Substituting this value of a and that of b given by equation 4 in equation 2 we have

$$D_{60+z} = 125 [1 + 4\sqrt{2}] [4\sqrt{2}]^z \quad \dots\dots\dots(8)$$

From equations 5 and 8 the relationship between Fineness Moduli and sizes of particles retained on different sieves can be shown to be

$$\log D_{60+z} = \left(\frac{1}{2} \log 2 \right) F_{60+z} + \log 125 [1 + 4\sqrt{2}] \quad \dots\dots\dots(9)$$

This relationship is given in Fig. 1.

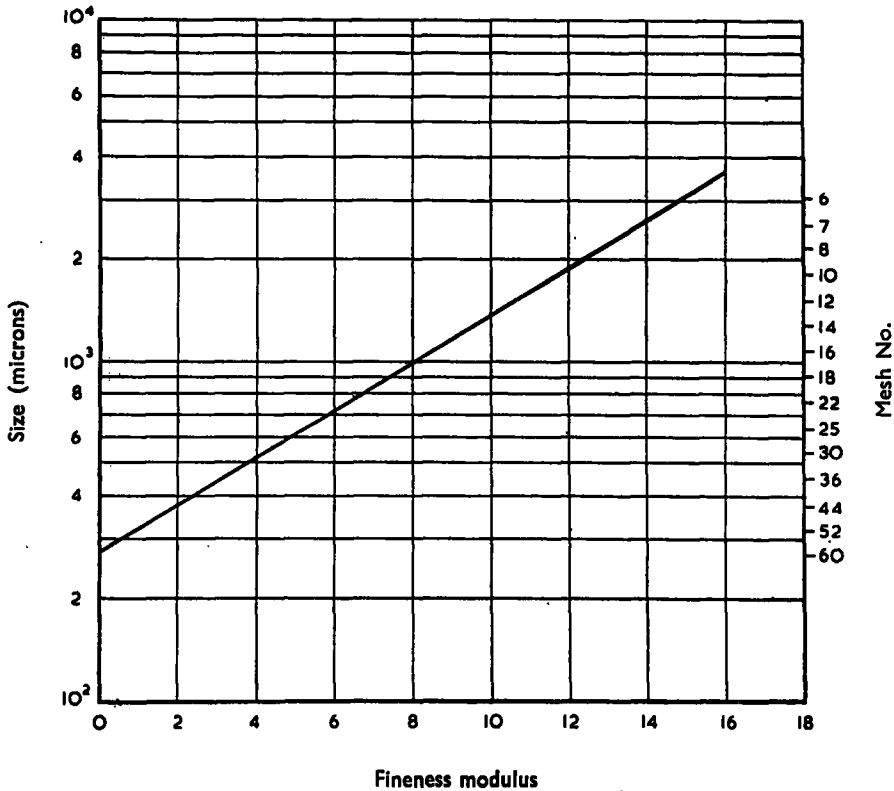


FIG. 1—Relationship between fineness modulus and particle sizes—BS mesh numbers are indicated for reference

Closer examination of equation 2 and the fact that sizes of particles in broken tea grades measured on an arithmetic scale are lognormally distributed, would reveal that particle sizes if measured on a scale of Fineness Moduli are normally distributed. Consider a polydisperse mixture of particles having y_{60+z} % particles, that would be retained on a mesh z steps larger than a mesh 60. The corresponding Fineness Modulus of these particles would be z .

It follows, therefore, that

$$\bar{z} = \frac{1}{100} \sum zy_{60+z} \dots\dots\dots(10)$$

and $\sum y_{60+z} = 100 \dots\dots\dots(11)$

\bar{D} , the geometric average size of particles is given by

$$\log \bar{D} = \frac{1}{100} \sum y_{60+z} \log D_{60+z} \dots\dots\dots(12)$$

Combinations of equations 2, 10 and 12 would yield

$$\log \bar{D} = \log a + \bar{z} \log b \dots\dots\dots(13)$$

In other words

$$\bar{D} = ab^{\bar{z}} \dots\dots\dots(14)$$

In broken tea grades since particle sizes as measured on a fineness modulus scale are normally distributed, the relationship between the expected per cent by weight retained on a sieve z steps larger than mesh No. 60 is

$$y_{60+z} = \frac{1}{s\sqrt{2\pi}} \int_z^{z+1} e^{-\frac{(z-\bar{z})^2}{2s^2}} dz \dots\dots\dots(15)$$

and the relationship between probit of cumulative per cent of sizes less than than corresponding to a value z (Finney 1947) is given by

$$Y_z = 5 + \frac{1}{s} (z - \bar{z}) \dots\dots\dots(16)$$

Equation 16 indicates a linear relationship between the probit of proportion undersize with respect to Fineness Modulus. Fractionating by means of B. S. test sieves would enable us to determine the proportion of undersize particles corresponding to different Fineness Moduli. Estimates of the two parameters \bar{z} and s which completely characterizes a grade could then be determined by regression analysis of probit undersize vs Fineness Moduli. Better estimates of these, however, could be obtained by Maximum Likelihood Solutions (Finney 1947).

Apparatus & Procedure

A sieve shaking machine and a complete set of 16 B.S. test sieves (BSS 410) supplied by Baird and Tatlock (London) Ltd., were used in sieve analysis. The diameter of each test sieve was 8 in. (20.3 cm) and the specifications are given in Appendix 2. The shaking machine was capable of accommodating eight test sieves. These were nested one above the other, so arranged that any one sieve had screen openings larger than the ones below. A solid pan (receiver) was placed under the bottom sieve. A lid was placed on the top-most sieve and the whole assembly was fixed tightly to the vibratory platform of the sieve shaking machine.

The sieve shaking machine was capable of vibrating the test sieves electromagnetically at 50 Hz and their movement combined vertical motion with a rotational action. This gave thorough stratification and caused the presentation of the particles at all angles to the sieve apertures. The shaking machine had a built-in control with a scale reading from 0 to 10 to vary the intensity of vibration, and a 0-to-60-min. time switch. A stop clock, however, was used instead of the built in timer, to determine intervals of shaking.

Preliminary experiments were directed to determine (a) a suitable sample size (b) a suitable period of shaking and (c) particle degradation, if any, during the sieving operations. It was possible to obtain more or less reproducible results with 100 g replicate samples having a common origin by adopting a period of 5 minutes for sieve shaking at the maximum intensity of vibration (corresponding to a scale reading of ten of the machine). With such a procedure, particle degradation during shaking was negligible. This standardized method of fractionating was adopted in all the experiments reported.

BOP and BOPF grades described as 'well sorted' 'too large in size', 'too small in size' and so on, were obtained from four tea brokers in Colombo. Duplicate samples of these obtained by successive splitting were fractionated into nine size ranges.

RESULTS

Summary of the results of sieve analyses of well-sorted samples supplied by the tea brokers in Colombo are presented in Tables 1 and 2. In Tables 3 and 4 are presented results of sieve analyses of samples described as 'not sorted correctly' by the tea trade.

Analysis

Provisional regression equations were first computed without attaching any weights to probits of percentages undersize, for each of the 36 samples investigated. These equations were subsequently used to obtain Maximum Likelihood Solutions. The correlation coefficients were highly significant ($P < 0.001$) for all samples of both BOP and BOPF, being greater than 0.9. These later solutions were utilized to obtain estimates of the parameters which completely describe the particle size distributions, viz, \bar{z} , the mean Fineness Modulus of the grades and s , the standard deviation of the distributions, of particle size as measured on a fineness modulus scale.

The maximum likelihood solutions obtained for the means of the samples given in Tables 1 and 2, taken to illustrate the computational procedure are—for BOP

$$Y_z = .685z - .518 \quad \dots\dots\dots(17)$$

and for BOPF

$$Y_z = .745z - .468 \quad \dots\dots\dots$$

TABLE 5 — *Values of mean Fineness Moduli and the standard deviations of particle size distributions as measured on a fineness modulus scale, of samples of BOP and BOPF described as not conforming to acceptable standards by the tea trade*

Grade	Sample No.	\bar{z}	$\frac{1}{s}$	s	Tasters Comments	Conclusion by Probit Analysis
BOP	1	7.418	0.657	1.522	not true to type too large in size too small in size	About the smallest tolerable size
	2	8.235	0.647	1.545		About the largest tolerable size
	3	8.345	0.599	1.669		About the largest tolerable size and range
	4	9.153	0.682	1.466		Too large in size
	5	7.944	0.877	1.140		Correct average size but range too narrow. This would give an appearance of smallness
BOPF	6	6.739	0.705	1.418	not true to type too large in size too small in size poor leaf and of good size —	Too large in size and about the largest tolerable range
	7	5.424	0.750	1.333		Too small in size
	8	6.224	0.707	1.414		Good size but about the largest tolerable range
	9	6.562	0.610	1.638		About the largest tolerable size and range too wide
	10	4.994	0.742	1.349		too small in size
	11	5.302	0.725	1.378		too small in size
	12	5.181	0.731	1.367		too small in size

SUMMARY AND CONCLUSIONS

The following conclusions could be drawn from the foregoing analysis and discussion.

1 — The particle sizes of high-grown BOP and BOPF grades are lognormally distributed. The mean of the logarithms of particle sizes and their standard deviation would, therefore, completely characterize the grades. Parameters which are related to these, *viz* the mean Fineness Modulus of particles and their standard deviation could be determined by sieve separation followed by probit analysis.

2 — Determination of parameters which characterize grades described as 'well sorted' have enabled to determine the 90% tolerance limits of these parameters. These limits are

$$\begin{aligned} & (1) \text{ for BOP grades} \\ & \quad 7.75 < z < 8.37 \\ & \quad 1.266 < s < 1.678 \end{aligned}$$

and

$$\begin{aligned} & (2) \text{ for BOPF grades} \\ & \quad 5.51 < \bar{z} < 6.65 \\ & \quad 1.199 < s < 1.437 \end{aligned}$$

3 — Values of parameters of grades described as being either oversize, or under-size by the tea trade fall outside the tolerance limits of the values of parameters determined for well-sorted grades. This criterion would help to decide whether a high-grown BOP or BOPF grade conforms to size standards that are generally acceptable. The same technique could be used, if desired, to determine other tolerance limits to suit individual requirements.

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Appendix 1

NOMENCLATURE

(A complete set of British Standard test sieves is implied where applicable).

- a = $\frac{1}{2}d_N [1 + 4\sqrt{2}]$ the average size of particles retained on a mesh having a nominal No. N (microns)
- b = $4\sqrt{2}$ (dimensionless constant)
- \bar{D} = Geometric average size of particles of a polydisperse mixture (microns)
- D_N = Average size of particles transmitted through a mesh one step larger than N and retained on N (microns)
- D_{N+z} = Average size of particles retained on a sieve z steps larger than N (microns)
- d_N = Average aperture size of sieve having a nominal No. N (microns)
- F_N = Fineness Modulus of particles retained on a sieve having a nominal No. N (dimensionless) defined by $F_{60} = 0$
 $F_{60+z} = z$
- F_{N+z} = Fineness Modulus of particles retained on a sieve z steps larger than a sieve having a nominal No. N, F_{60} being taken as zero
- g = Abbreviation for grams
- N = Nominal Mesh No. (dimensionless)
- Y_z = Probit of cumulative percent of sizes less than a size corresponding to a Fineness Modulus z (dimensionless).
- y_{60} = Per cent by weight of particles retained on Mesh No. 60 (dimensionless)
- y_{60+z} = Percent by weight of particles retained on a mesh z steps larger than mesh No. 60 (dimensionless)
- z = Fineness Modulus of particles retained on a sieve z steps larger than a sieve having a nominal No. 60 (dimensionless)
- \bar{z} = Mean Fineness Modulus of a polydisperse mixture of particles (dimensionless)
- s = Standard deviation of normally distributed particle sizes of tea grades as measured on a Fineness Modulus scale.

In addition to these, other standard mathematical symbols have been adopted.

Appendix 2

SPECIFICATIONS OF TEST SIEVES

A complete set of test sieves conforming to BS. 410 (1969) having a ratio between successive sieves equal to $4\sqrt{2}$ or 1.19, was used.

Nominal Mesh No.	Aperture width (microns)	Mean size of particles retained (microns)	Fineness Modulus of particles retained
5	3350	—	—
6	2800	3075	14
7	2400	2600	13
8	2000	2200	12
10	1680	1840	11
12	1400	1540	10
14	1200	1300	9
16	1000	1100	8
18	850	925	7
22	710	780	6
25	600	655	5
30	500	550	4
36	420	460	3
44	355	388	2
52	300	328	1
60	250	275	0