

Modeling Trough Withering System to Predict the Moisture Content of Tea Leaves at Real Time using One Dimensional Heat and Mass Transfer Finite Difference Model

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ABSTRACT

One-dimensional heat and mass transfer mathematical model was developed to predict moisture content of tea leaves during trough withering. Model solutions were approached using finite difference method by giving appropriate initial and boundary conditions. A computer program was written to calculate the real time moisture content of tea leaves and other psychometric parameters of conditioned air during withering. Six experiments were performed to validate the developed model using commercial type of withering trough. Leaf samples were drawn from top, middle and bottom layers of the leaf bed in the trough for one-hour time interval for 12h of the test period. Moisture data calculated by the program was compared with the experimental data. Time and space increments of the model were chosen minimizing the estimation error of moisture content. Results showed that the experimental and simulated average moisture content were in close agreement with standard error in the range of 0.63 – 1.23 on percent wet basis.

Keywords: Fresh tea leaves, one dimensional heat and mass transfer model, withering trough

INTRODUCTION

Tea is a popular beverage throughout the world because of its astringent taste and inherent flavor characteristics. The primary goal of tea industry is to supply the market with safe and quality product. Quality of made tea is directly influenced by the taste and aroma of tea liquor. Good manufacturing practices are necessary to process good quality tea. Black tea processing involves five major unit operations namely withering, rolling, fermentation, drying and grading. Withering is very important in black tea processing because it consumes the highest amount of electrical energy and fairly large amount of thermal energy. In addition, uniformly withered tea leaves are essential to produce good quality end product.

Initial moisture content of fresh tea leaves varies from 70 – 83% w.b. depending on the climatic condition, weather pattern and the type of tea cultivar (Samaraweera, 1986). Orthodox is the most popular black tea processing method in Sri Lanka in which 70 – 83% moisture content of tea leaves is reduced to 55% w.b. within 12 - 16h during withering. The appearance of tea shoots at the final stage of withering has pale green color and flaccid with unbreakable stems. A person with good experience can identify the correct stage of withering. Therefore determining correct withering stage is merely depends on the personal experience. The moisture content of tea leaves in withering determines the wither percentage. Therefore different methods and meters were developed to determine the moisture content of leaf samples. The standard method of determining moisture content of tea leaves is using air oven for 6h at 103°C (ISO 1568, 1980). A method was later developed to determine moisture content of withered leaf within 10min period using microwave oven (Mohamed *et al.*, 2003). Standard method is not practicable to determine the real time moisture content during withering whereas microwave oven method is cumbersome to conduct.

Different mathematical models with one/two dimensional heat and mass transfer equations were used to predict real time temperature, moisture content *etc.* in various food products (Muir, *et al.*, 1980; Murata *et al.*, 1996). The numerical solutions of these models can be obtained by finite difference method (Yacink, *et al.*, 1975). In the literature, it is difficult to find any developed mathematical approach to determine moisture content of tea leaves during withering.

In this study, one-dimensional heat and mass transfer model was developed for tea withering and the numerical solutions of the model were approached by finite difference method. The model was validated using a commercial type of withering trough.

Model development and numerical solutions

Heat and mass transfer phenomenon between conditioned air and tea leaf surface was considered in developing the model for a deep bed of tea leaves in the trough. Both the thermal and the flow properties of air together with geometry of the system are directly influenced to heat and mass transfer coefficients. The set of differential equations was used to describe the withering process of deep bed of tea leaves under the following assumptions.

- Withering process was assumed an adiabatic and reversible.
- Heat transfer by conduction between leaves was assumed negligible.
- There was no hysteresis effect between adsorption and desorption isotherms of tea leaves.
- The temperature and relative humidity distribution in the leaf bed were uniform in horizontal direction, *i.e.* heat and moisture flow in the horizontal direction was assumed negligible.

Based on these assumptions, only the vertical component of the airflow was considered in model development. Therefore the problem was treated as one-dimensional process.

Model development of trough withering

The model contains differential equations for calculating leaf temperature, leaf moisture, enthalpy and absolute humidity of air at an arbitrary point in the deep bed. Leaf bed was divided into finite number of thin layers in modeling (Figure 1). Air enters into the bed of leaf from the bottom and flows across the bed and leaves at the top of the bed.

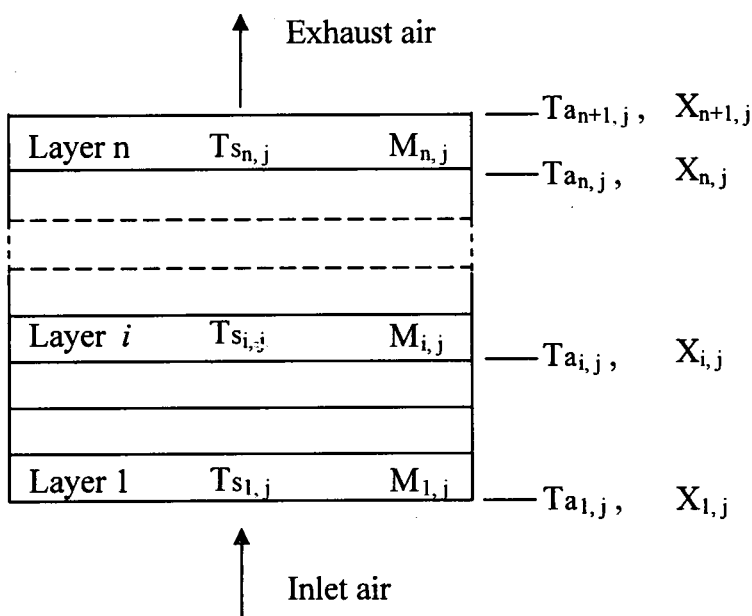


Figure 1. Finite difference scheme

The leaf bed was divided into finite number of horizontal thin layers of tea leaves as shown in Figure 1. Each layer was subjected to uniform conditions of air throughout the selected time interval, Δt . Exhaust air conditions from one layer were taken as input conditions to the layer immediately above. Knowing the inlet air conditions at any given layer and the initial layer moisture content, final air conditions and leaf moisture content were calculated at the end of Δt . The procedure was repeated for each layer in the bed at a given time interval.

Following equations were used to model the withering process of tea leaves.

The drying rate of fresh tea leaves is calculated using the following differential equation.

$$\frac{dM}{dt} = -k(M - M_e) \quad (1)$$

Where k is the drying coefficient of fresh tea leaves and was calculated using the equation (2) with temperature and relative humidity of air (Botheju *et al.*, 2008).

Drying coefficient (k):

$$k = f_1 \exp\left(\frac{f_2 h^3 + f_3 h^2 + f_4 h + f_5}{T_a}\right) \quad (2)$$

Where;

$$f_1 = 728705.0448,$$

$$f_2 = 219.5551,$$

$$f_3 = -1998.3448,$$

$$f_4 = 1146.5436,$$

$$f_5 = -6121.8327.$$

Equilibrium moisture content (M_e):

M_e , the equilibrium moisture content in equation (1) was determined by Oswin isotherm equation (equation 3). Parameters were calculated for fresh tea leaves.

$$M_e = (55.4994 - 0.1272T_a) \left[\frac{h}{1-h} \right]^{0.2275} \quad (3)$$

Finite difference solution of the equation (1) with initial conditions of $M = M_0$ to calculate the moisture content of i^{th} layer, M_i of withering tea leaves is given by equation (4).

$$M_{i,j+1} = M_{i,j} - k(M_{i,j} - M_{ei,j})\Delta t \quad (4)$$

Leaf temperature at the beginning of withering is very close to wet bulb temperature of air. Murata, *et al.* 1996 has used equation (5) for calculating the material temperature in deep bed simulation and the same equation was used in calculating the leaf temperature in withering.

$$\frac{\partial T_s}{\partial t} = \left(\frac{h_a}{C_{ps}\rho_s} \right) (T_a - T_s) - \left(\frac{kq}{C_{ps}} \right) \frac{(M - M_e)}{100} \quad (5)$$

The 1st term of equation (5) denotes rate of leaf temperature rise whilst the 2nd term indicates rate of water temperature increase in the leaf due to the rise of leaf temperature.

Where h_a is the convective heat transfer coefficient and was calculated using equation (6) (Botheju *et al.*, 2009).

$$h_a = 0.0025G \quad (6)$$

Heat of evaporation (q):

The heat of evaporation of tea leaves decreases with increasing moisture content in the desorption process. Rearranging the Clausius–Clapeyron equation (equation 7, Moore, 1962) gives the change of saturated vapor pressure with q .

$$q = T(V_g - V_l) \frac{dP_{st}}{dT} \quad (7)$$

The term $(V_g - V_l)$ was calculated by equation (8).

$$V_g - V_l = R_w \frac{T}{p} - 0.001 \quad (8)$$

Where $R_w = 0.4619$ kJ/kg K (Cengel and Boles, 1989).

The vapor pressure in tea leaves, P_{st} was calculated using equation (9).

$$P_{st} = hP \quad (9)$$

dP_{st}/dT in equation (7) was obtained by differentiating equation (9) with respect to T given by equation (10),

$$\frac{dP_{st}}{dT} = \frac{dh}{dT} P_s + \frac{dP_s}{dT} h \quad (10)$$

The term dh/dT can be calculated by differentiating the equation (3) with respect to T . dP_s/dT is calculated by differentiating the equation (11), the psychometric properties of moist air P_s (Bolton, 1980).

$$P_s = 0.61078 \exp \left(\frac{T_c}{(T_c + 238.3)} * 17.2694 \right) \quad (11)$$

Substituting all the terms into equation (7), q can be calculated.

Bulk density (ρ_s):

The bulk density of green leaves of the trough bed varied depending on the quantity of leaf received to the factory. When withering is progressed, Δ_s was significantly varied with time and followed equation (12).

$$\rho_s = \frac{W_t}{V_t} \quad (12)$$

$$\text{Where } W_t = (100 - M_o) \left[\frac{W}{(100 - M_{wb})} \right] \quad (12.1)$$

Where W is the weight of leaves loaded to the trough and V_t is the volume of the trough.

Specific heat capacity (C_{ps}):

The specific heat capacity (C_{ps}) of tea leaves was determined using equation (13) (<http://www.bae.uky>).

$$C_{ps} = 0.827 + 3.348M_{d(wb)} \quad (13)$$

Finite difference solution of equation (5) to calculate the leaf temperature could be given as follows (equation 14).

$$T_{si,j+1} = T_{si,j} + \left(\frac{h_a}{C_{ps}\rho_s} \right) (T_{ai,j} - T_{si,j}) \Delta t - \left(\frac{kq}{C_{ps}} \right) \left(\frac{M_{i,j} - M_{ei,j}}{100} \right) \Delta t \quad (14)$$

The absolute humidity of air in the leaf bed was calculated by equation (15) (Murata, *et al.* 1996).

$$\frac{\partial X}{\partial y} = K_2 (M - M_e) / 100 \quad (15)$$

$$\text{Where; } K_2 = \rho_s \frac{k}{G} \quad (16)$$

$$G = V\rho_a \quad (17)$$

$$\rho_a = \frac{353.3}{T_a} \quad (18)$$

Numerical solution of the equation (15) to calculate absolute humidity (kg-water/kg dry air) of air is in the following form (equation 19);

$$X_{i+1,j} = X_{i,j} + K_2 (M_{i,j} - M_{e,i,j}) \Delta l \quad (19)$$

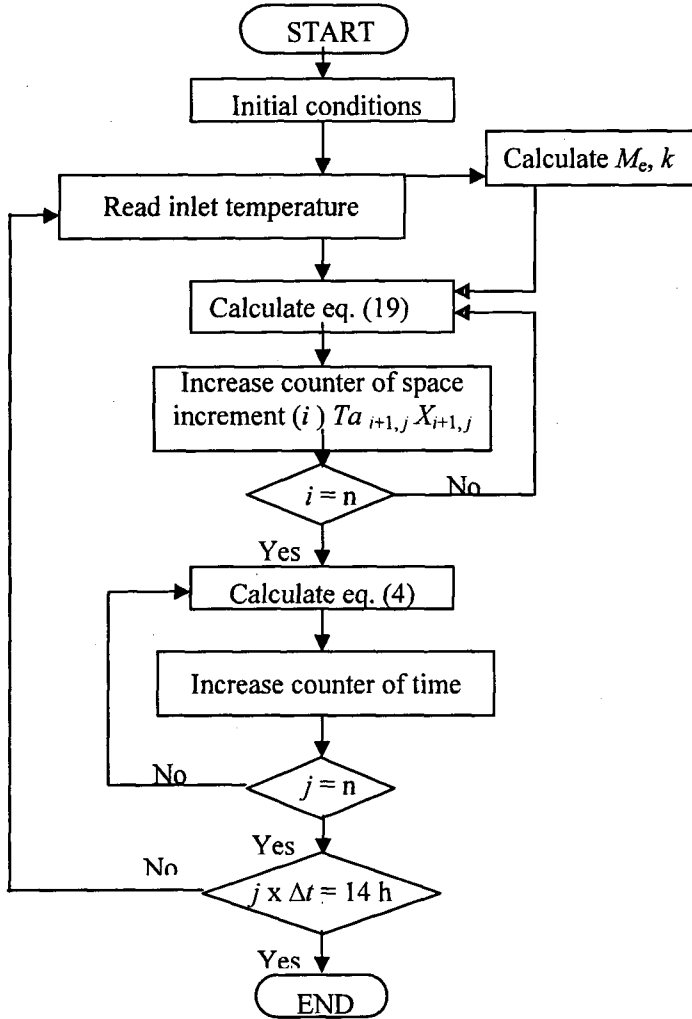


Figure 2. Program flow chart for predicting moisture content of tea leaves during withering

The governing equation to calculate enthalpy of the conditioned air is given by equation (20) (Murata, *et al.* 1996).

$$\frac{\partial I}{\partial y} = (\sigma K_2)(M - M_e) - \left(\frac{h_a}{G}\right)(T_a - T_s) \quad (20)$$

Where is very close to the latent heat of vaporization of pure water.

The solution of the equation (20) in the finite difference form could be written as (equation 21):

$$I_{i+1,j} = I_{i,j} + (\sigma K_2)(M_{i,j} - M_{ei,j})\Delta l - \left(\frac{h_a}{G}\right)(T_{ai,j} - T_{si,j})\Delta l \quad (21)$$

Air temperature (T_a) changes across the leaf bed was calculated using equation (22).

$$T_a = \frac{I + 0.026 - \sigma X}{C_{pa} + C_{pw}X} \quad (22)$$

The relative humidity of air (rh) leaving the layer was calculated by equation (23).

$$rh = \frac{P}{P_s} \quad (23)$$

Where;

$$P = \frac{PX}{0.622 + X} \quad (24)$$

Atmospheric pressure, $P = 101.325$ kPa. Saturated vapor pressure P_s was calculated using equation (11).

In the model, Δl element thickness varied while withering was progressed. An empirical relationship was for leaf bed height was developed and incorporated with the model and $\Delta t = 5.5$ s. The initial and boundary conditions are shown below.

Initial conditions $T_{si,j}$ = Wet bulb temperature

$M_{i,j}$ = Initial moisture content of tea leaves

Boundary conditions $T_{ai,j}$ = Air temperature measured during withering

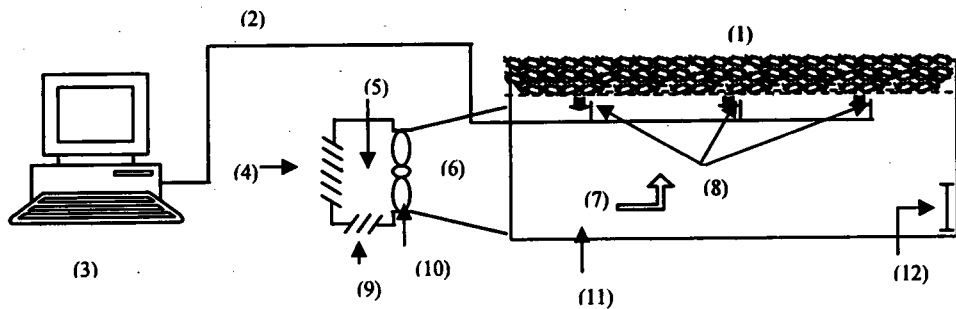
$X_{i,j}$ = Relative humidity of air during withering

QBASIC program was written based on the flow chart shown in Figure 2. Velocity of air decreased during the withering process. A quadratic relationship was developed between superficial air velocity and the moisture content.

METHODOLOGY

Model validation

Tea leaves plucked in the morning from St. Coombs estate were transported to the tea factory and weighed at ± 1 kg accuracy. The commercial scale withering trough with the area of 28.5 m^2 was loaded with 800 kg of tea leaves. Pre-calibrated temperature and RH sensors coupled with PC-based data acquisition system were fixed at three positions in the trough chamber to measure temperature and RH of air (Figure 3) entering to the trough chamber during withering. Random sampling was conducted from front, center and rear end of the trough bed before switching on the trough fan to determine the initial leaf moisture. Fan was switched on and leaves were immediately loosened. The operation of data acquisition system was commenced simultaneously to read the RH and temperature of air in the plenum chamber throughout withering. Wet bulb temperature of air in the chamber was also recorded.



- | | |
|-----------------------------|--------------------------------|
| (1) Leaf bed | (7) Axial fan |
| (2) Wire connection | (8) Airflow path |
| (3) Data acquisition system | (9) Temperature and RH sensors |
| (4) Cold air | (10) Warm air |
| (5) Mixing chamber | (11) Plenum chamber |
| (6) Transformation duct | (12) Gable door |

Figure 3. Schematic diagram of the experimental trough withering set up

After 30 min of switching on the fan, 18 samples were drawn from top, middle and bottom layers of the leaf bed along the length of trough and immediately put them into a polythene bag and sealed. Known quantity of sample (50 g) was then drawn from each bag to determine the moisture content in three different layers using an air oven method (103°C for 6h).

The sampling procedure was repeated for every one-hour time interval throughout withering. Initial moisture content and wet bulb temperature were fed to the QBASIC

program. Air temperature and RH measured from the sensors were also given to the program as boundary values. Moisture contents of top, middle and bottom layers and the average moisture content calculated by the model were compared with the experimental moisture data.

RESULTS AND DISCUSSION

The heat of vaporization (q) of water in tea leaves was calculated using the equation (7). 'q' versus moisture content of tea leaves at four different temperatures are presented in Figure 4.

It shows that heat of vaporization decreases with increasing of the moisture content. The value of 'q' approached the latent heat of free water at the moisture content above 40% d.b. Decreasing moisture content below 20% d.b. heat of vaporization increased almost exponentially. This is because of the water is bound tightly at lower moisture contents.

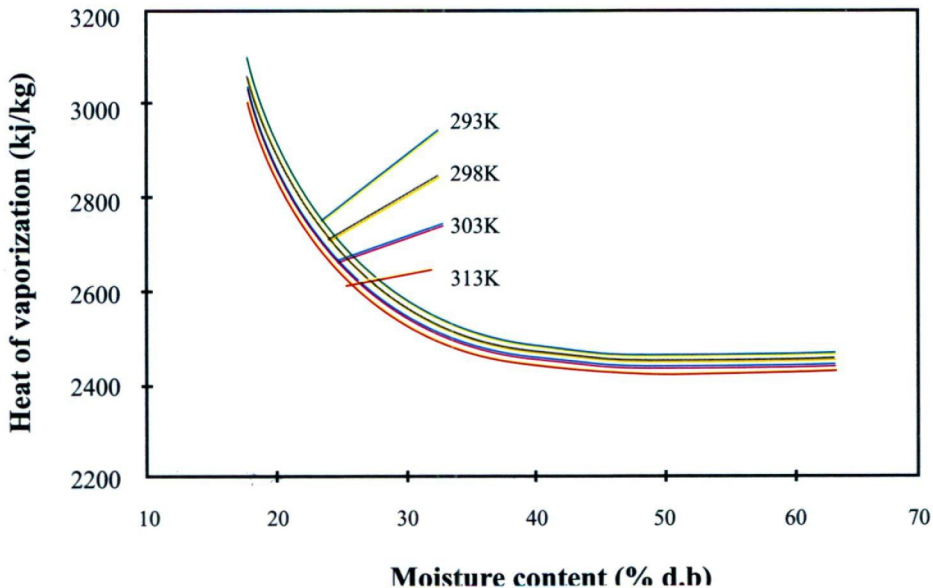


Figure 4. Heat of vaporization of water in tea leaves at four different temperatures and moisture contents

Initial temperature and RH of air entering to the withering trough were measured using the data acquisition system and the variation of the parameters with time is given in Figure 5.

The graph shows that withering air temperature was gradually increased for about 1/2h due to supply of hot air after about 2 1/2 h time of withering. Just after completing the drying, the hot air was supplied to remove some surface moisture on the leaves. As a result of increasing the air temperature, RH was decreased.

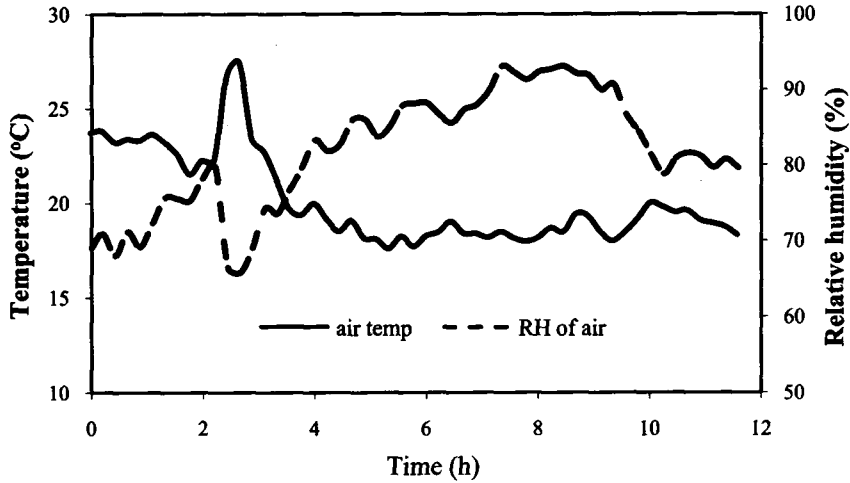


Figure 5. Variation of temperature and RH of air during the test period of withering in experiment (I)

Results of two validation trials were presented in Figure 6. The average moisture contents calculated from the samples drawn from three different layers were compared with the model calculated average moisture data for 12h withering period. The model calculated average moisture contents of tea leaves were calculated by the QBASIC program. Graphs show that model calculated moisture data were in close agreement with the experimental data.

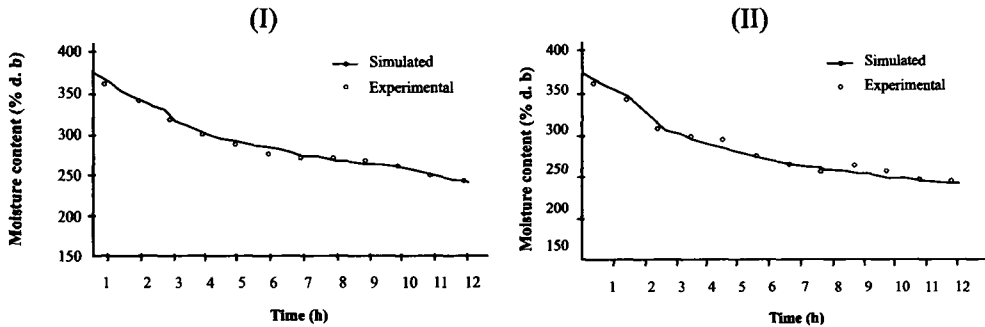


Figure 6. Average experimental and simulated moisture contents of three different layers of experiments (I) and (II)

Statistical criteria to evaluate the correlation of experimental and simulated curves are given in Table 1. It reveals that the average moisture content of observed data showed a good agreement with the simulated data with standard error (*SE*) in the range of 0.6248 – 1.2319 and mean relative deviation (*P*) in the range of 0.6742 – 1.4950 on percent wet basis.

Table 1. Standard error and mean relative deviation of average moisture content of tea leaves

Experiment No	<i>SE</i>	<i>P (%)</i>
1	0.6248	0.6742
2	1.0581	1.3121
3	1.2319	1.4950
4	0.4797	0.5241

The Figure 7 demonstrates the variation of experimental and simulated moisture contents of tea leaves on top, middle and bottom layers of the leaf bed. A slight deviation of the simulated curves was noticed in the graphs of bottom layers at the latter stage of the withering process. Experimental moisture was higher than simulated value of the bottom layer of the leaves whilst the top or middle layer indicates higher moisture content in predicted data than the experimental data at the latter stage of withering. This deviation is due to reducing the bed thickness of the leaf layer which mainly effect to the superficial air velocity at the bottom of the leaf bed due to caking the bottom leaf layer. Therefore air can escape from loosely packed localized spots in the withering trough at the latter stages of withering.

CONCLUSIONS

One-dimensional heat and mass transfer model was developed to simulate the real time moisture content of tea leaves in trough withering. A commercial type of withering trough was used to validate the developed model. Results showed that the model calculated moisture data were in good agreement with the experimental data. Average moisture content of observed data showed a good agreement with the simulated data with standard error (*SE*) in the range of 0.6248 – 1.2319 on percent wet basis.

Therefore the model can be successfully used to predict moisture content of tea leaves during withering.

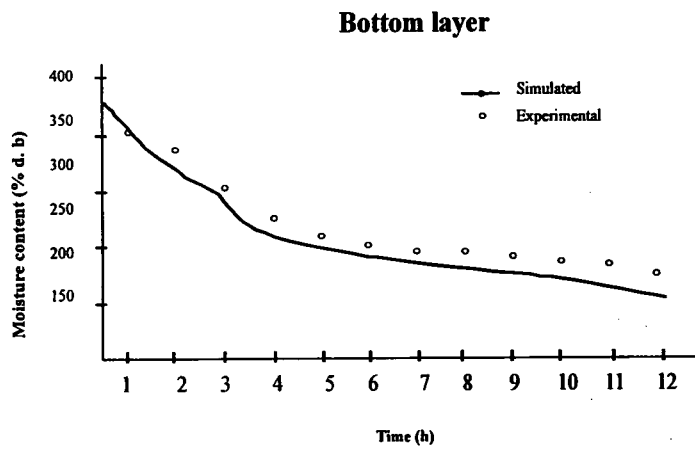
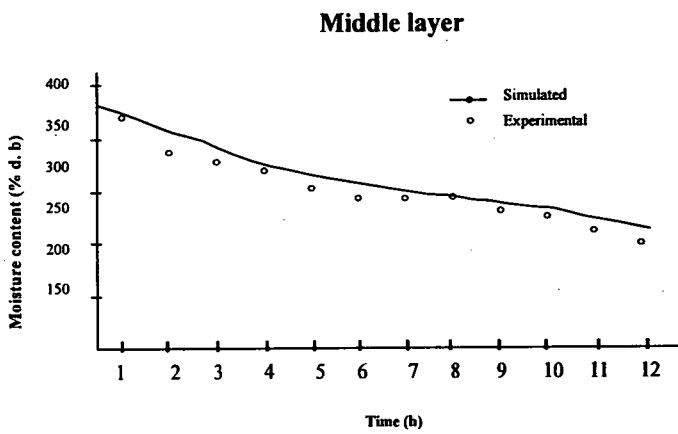
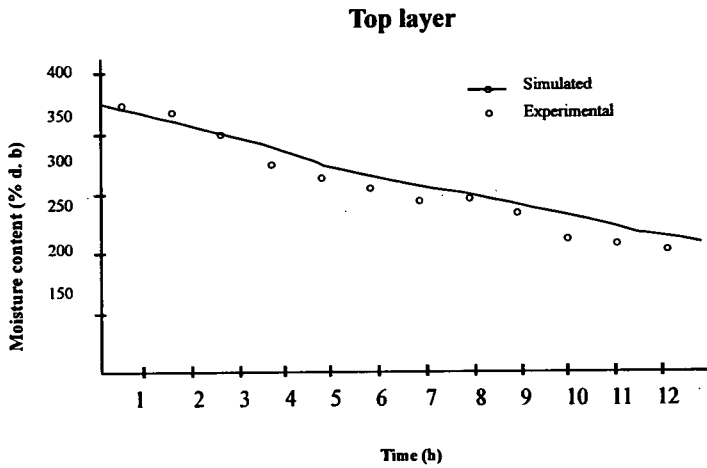


Figure 7. Experimental and simulated moisture contents of top, middle and bottom layers in one experiment.

REFERENCES

- Bolton D 1980 The computation of equivalent potential temperature. *Monthly Weather Review*. 108(7), 1046 - 1053.
- Botheju W S, Amaratunga K S P and Abeysinghe I S B 2008 Thin layer drying characteristics of fresh tea leaves. *Proceedings of the 2nd Symposium on Plantation Crop Research, BMICH, Colombo, Sri Lanka, 2008; 10 - 19.*
- Botheju W S 2009 Heat transfer coefficient of fresh tea leaves: PhD thesis titled "Moisture simulation of tea leaves in trough withering using finite element heat and mass transfer model", 110.
- Cengel Y A and Boles M A 1989 Molar mass, gas constant and critical-point properties. *In Thermodynamics and Engineering Approach 2nd edition. McGraw-Hill, pp. 868.*
- Internet Document URL www.bae.uky.edu/snokes/BAE549thermo/physicalproperties/thermalprops.html. Accessed on 29/03/2007.
- ISO 1568 1980 Tea – Determination of loss in mass at 103°C. International Standard Organization Second Edition 1573-1980 (E), Switzerland.
- Mohamed M T Z, Raveendran K, Botheju W S, Priyanthi S H and Jayasinghe L 2003 A rapid method to determine moisture content in green leaf, withered leaves and made tea using a microwave oven. *Sri Lanka Journal of Tea Science* 68(2), 44-48.
- Morey R V, Keener H M, Thompson T L, White G M and Bakker-Arkema F W 1978 The present state of grain drying simulation. *ASAE Paper No. 78-3009.*
- Muir W E, Fraser M and Sinha R N 1980 Simulation model of two dimensional heat and mass transfer in controlled-atmosphere grain bins. *In Controlled Atmosphere Storage Grain Ed. J Shejbal, Amsterdam. Elsevier Scientific Pub. Co. 385-398.*
- Murata S, Amaratunga K S P, Tanaka F, Shibuya K and Koide S 1996 Simulation of moisture adsorption by polished rice in deep-bed. *Food Science & Technology Institute*, 2(2), 86 - 91.
- Samaraweera D S A 1986 Technology of tea processing *In Handbook on Tea Eds. P Sivapalan, S Kulasegaram and A Kathiravetpillai. Tea Research Institute of Sri Lanka. 158-207.*

Yaciuk G, Muir W E, Fraser M and Sinha R N 1975 A simulation model of temperatures in stored grain. Journal of Agriculture Engineering Research, 20, 245-258.

Nomenclature

C_{ps}	Specific heat capacity of tea leaves (kJ/kg.K)
C_{pa}	Specific heat capacity of air (kJ/kg.K)
C_{pw}	Specific heat of wet air (kJ/kg.K)
d.b.	Dry basis
G	Mass flow rate of air (kg/s)
h_a	Heat transfer coefficient of tea leaves (kJ/min.m ² .K)
h	Equilibrium relative humidity (decimal)
I	Enthalpy of air (kJ/kg)
k	Drying constant / coefficient (sec ⁻¹)
M_e	Equilibrium moisture content (% d.b.)
M	Moisture content of tea leaves at given time (% d.b.)
$M_{d(wb)}$	Moisture content of tea leaves in decimal wet basis
P_s	Saturated vapor pressure of water (Pa)
P_{st}	Vapor pressure of tea leaves (Pa)
q	Latent heat of evaporation of tea leaves (kJ/kg)
R_w	Gas constant of water vapor (kJ/kg K)
rh	Relative humidity (decimal)
T_a	Air temperature (K)
T_c	Temperature (°C)
T_s	Temperature of tea leaves (K)
t	Time (sec)
V_g, V_l	Specific volume of water vapor and liquid water at given temperature (m ³ /kg)
V	Volume flow rate (m ³ /sec)
w.b.	Wet basis
X	Absolute humidity (kg water/kg of air)
y	Layer thickness
Δ_s	Bulk density of tea leaves (kg/m ³)
Δ_a	Density of air (kg/m ³)
Δ	Latent heat of vaporization of water (kJ/kg)
Δl	Element thickness (m)
Δt	Time increment (sec)