



INSTITUTE OF PHYSICS – SRI LANKA

**Research Article**

**Impact of Coronal Mass Ejection on different characteristic parameters  
of lower thermospheric region**

**Suman Paul<sup>1,2\*</sup> & S.S. De<sup>1</sup>**

<sup>1</sup>*Centre of Advanced Study in Radio Physics and Electronics, University of Calcutta,  
Kolkata 700009, INDIA*

<sup>2</sup>*Department of Physics, Rishi Bankim Chandra Evening College, Naihati,  
24 Parganas (N), PIN 743165, INDIA*

---

**Abstract**

Protons, electrons, alpha particles, and heavier ions in the solar wind are emitted from the Sun's coronal plasma. Energetic phenomenon such as a Coronal Mass Ejection (CME) releases high energy as electromagnetic radiation in the form of solar X-rays and  $\gamma$ -ray bursts. This intense radiation is responsible for ionizing the constituent particles of the various layers of Earth's ionosphere. In the presence of perturbations of the thermospheric region produced by CME, different characteristic parameters are theoretically investigated through momentum and energy balance equations, equation of continuity and equation of state. Dispersive nature of the medium is investigated and altitude variations of resulting electric field are carried out. In the computation processes, data of the latest International Reference Ionosphere (IRI) models are used. The results obtained are to be compared and supported by the experimental findings from other researchers to understand the nature of the medium in detail.

**Keywords:** Coronal mass ejection, Dispersion, International Reference Ionosphere, Magnetic reconnection, Thermosphere

---

\* Corresponding Author Email: paul\_suman30@yahoo.co.in

 <http://orcid.org/0000-0002-6757-3992>

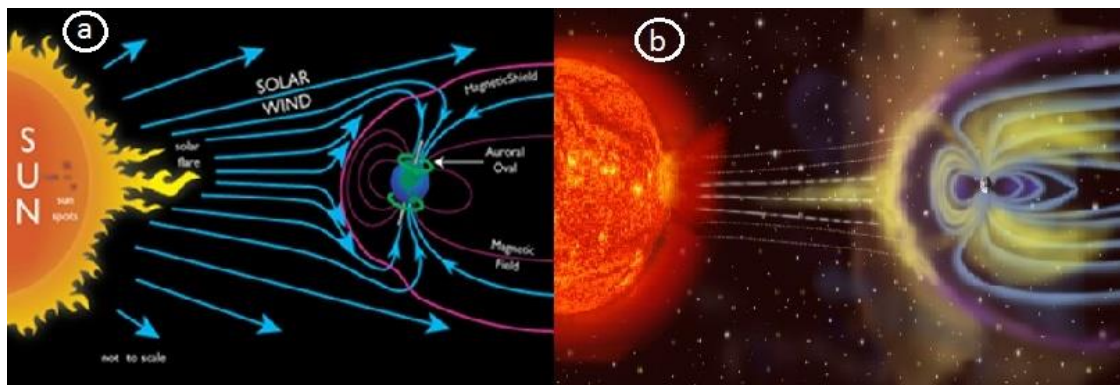


## 1. INTRODUCTION

The solar wind is a continuous phenomenon occurring from the solar coronal zone as an outflow of solar plasma, viz., protons, electrons, alpha particles, trace amounts of heavier ions<sup>1-7</sup>. CMEs are one of the most dynamic and energetic aspects of solar activity that cause a rotation of the magnetic field and mostly originate from solar active regions. It interacts with the magnetosphere of the Earth producing perturbation in the Earth's ionosphere. CMEs and solar flares are the key processes associated with magnetic reconnection<sup>8</sup>. In the process, two oppositely directed fields intersect releasing energy that stored in the originally stressed magnetic fields. Also, CMEs play a major role in the generation of interplanetary disturbances and can disrupt the magnetic and particle environment near Earth causing space weather. Solar wind creates and maintains the bubble like regions, i.e., heliosphere in the Sun against the outside pressure of the interstellar medium. From the Sun, the solar wind flows outward till it encounters the termination shocks. The interaction between the solar wind and Earth's magnetic field produces the magnetosphere of the Earth (cavity) and interaction of solar wind and interstellar medium produce numerous bubbles in space known as magnetosphere of the Sun.

In the process of magnetic reconnection, the magnetic energy is transformed into kinetic and thermal energy of electrons and ions which allow the heat flux to drive the instabilities satisfying the time averaged energy requirements of the solar wind. The ion-temperature and the coronal holes is proportional to its mass, hence heavier ions can have greater flow velocities than protons and the coronal holes at the site<sup>7,9</sup>. The solar wind is a near-continuous flow of coronal material. A transient event, CME, is a cloud of magnetized solar material that erupts from the solar corona into interplanetary space, and is often associated with flares. Figure 1 panel (a) schematically shows the interaction of solar wind with the Earth's atmosphere through the magnetopause while panel (b) shows the eruptions of magnetized solar material during CMEs. The solar atmosphere consists of very high temperature plasma and is fully dynamic in nature. Due to its high dynamic activity, large numbers of magnetic dipoles are continuously generated with intense magnetic fields. The energetic charged particles released from the solar corona are trapped within the dipole field. Those particles will gyrate along the magnetic field lines and accumulate near sunspots. These field lines are gradually elongated outwards from the Sun due to the radial momentum of the massive charged particles trapped in the field ultimately causing breakdown in the

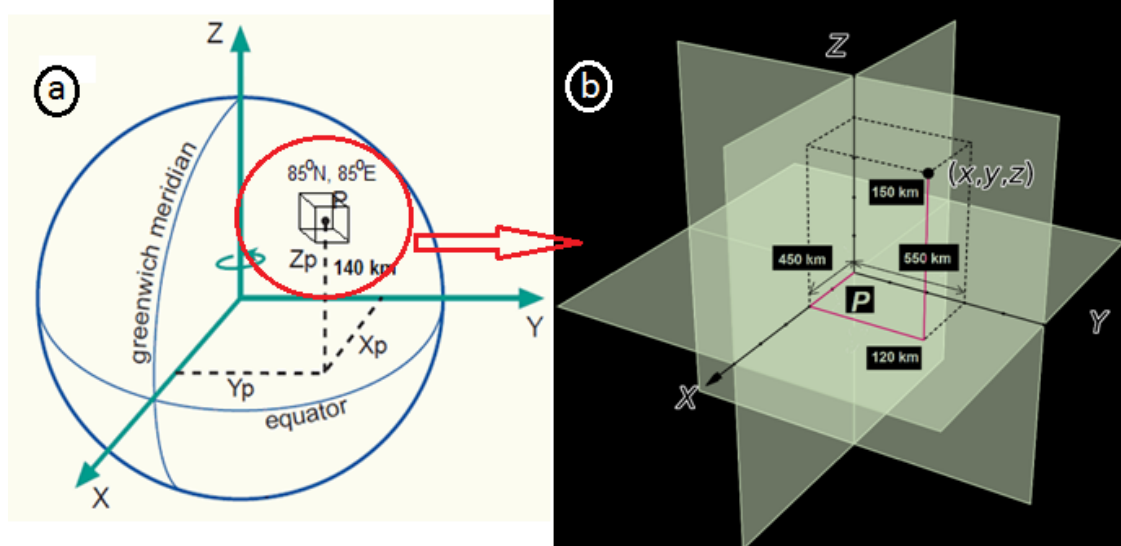
field lines at their critical limit, thus establishing magnetic reconnection with the interplanetary magnetic field. This process generates and releases trapped particles and electromagnetic radiation in the form of solar X-rays and  $\gamma$ -ray bursts<sup>10-14</sup>. This intense radiation is responsible for ionizing the constituent particles of the ionosphere. The region is thereby highly modified and thus, the characteristic behaviour of the Earth-ionosphere waveguide is abruptly changed.



**Figure 1:** (a) Interaction of solar wind with the Earth's atmosphere through the magnetopause. (b) Eruptions of magnetized solar material during CMEs.

In this paper, the situation has been investigated by momentum balance equation, energy balance equation along with the equations of continuity and states. The data of International Reference Ionosphere (IRI-2007)<sup>15</sup> based on Virtual Ionosphere, Thermosphere, Mesosphere Observatory (VITMO) model is used in the computation. For geographic latitude at  $85^\circ$  N and longitude  $85^\circ$  E, and at average altitude of 140 km from the surface of the Earth (Figure 2a), the values of the electron number density, electron temperature, wave propagation factor, and plasma frequency are taken from this model. The ionospheric region perturbed by the CMEs is schematically represented in Figure 2a. The region of interest (red circle) is magnified and shown in Figure 2b with the local Cartesian co-ordinate system and bounding dimensions (Note: We neither employ any finite element model to follow propagation nor implement the equations into a numerical algorithm). The volume is treated as stationary with a flow of material through the bounded region. MATLAB programs are used to calculate the values of complex refractive indices of the

medium along with the real (refractive) and imaginary (absorption index) part as well as electric fields within the medium for these altitudes. The output of our program is plotted through Origin 5.0.



**Figure 2:** (a) Schematic diagram showing the co-ordinate system used in this problem. The frame of reference is on the surface of the Earth. (b) Dimensions of the volume of the ionosphere perturbed by CME for finite computation of the parameters addressed in this problem.

## 2. MATHEMATICAL FORMULATIONS

The physical situation stated above can be represented by the following energy balance equation, equation of continuity and momentum balance equations<sup>16-18</sup>: We consider a self generating electromagnetic wave that moves along the X-direction in the lower thermosphere. Due to the effect of collisions of electrons with other electrons and gas phase atoms, there will be a frictional force which is in general proportional to velocity. Other aspects will be energy transfer from one electron to other atoms and transfer due to electron drift also. Other terms in energy balance equation relate to thermal conductivity and ionization.  $e(\vec{E} + \vec{v} \times \vec{H})$  is the Lorentz force for an electromagnetic field. In the equation of continuity, the terms pertain to the excess electrons from two competing phenomena, i.e., ionization and electron-neutral attachment. The loss term relates to electron-ion

recombination. Besides this, in momentum balance equation, collisions with heavy particles and viscosity terms are considered.

$$\frac{3}{2} \frac{\partial}{\partial t} (N_e k T_e) + e N_e \bar{v} \cdot (\bar{E} + \bar{v} \times \bar{H}) + G_{eff}(T_e) \nu_e(T_e) - \nabla \cdot \bar{q} - \chi \nabla^2 T_e + Q_i \frac{\partial N_e}{\partial t} = 0 \quad (1)$$

$$\frac{\partial N_e}{\partial t} = (\nu_i - \nu_a) N_e - \alpha N_e^2 \quad (2)$$

$$m \frac{\partial \bar{v}}{\partial t} + m (\bar{v} \cdot \nabla) \bar{v} = -e \bar{E} - m \nu_e(T_e) \bar{v} - e (\bar{v} \times \bar{H}) - b \bar{v} + \eta \nabla^2 \bar{v} \quad (3)$$

Where,  $N_e$ , the number of electrons per unit volume;  $k$ , the Boltzmann constant;  $T_e$ , the electron temperature;  $e$ , charge of an electron;  $\bar{v}$ , velocity vector of an electron;  $\bar{E}$ , the electric field vector;  $\bar{H}$ , the geomagnetic field vector;  $G_{eff}(T_e)$ , effective fraction of energy transfer per collision;  $\nu_e(T_e)$ , effective collision frequency of electrons with heavy particles;  $\bar{q}$ , total energy flow due to electron drift when the heavy neutral and ion components are assumed to have no-net drift velocity;  $\chi$ , thermal conductivity;  $Q_i$ , ionization energy of the medium;  $\nu_i$ , ionization frequency;  $\nu_a$ , electron neutral attachment frequency;  $\alpha$ , electron-ion recombination coefficient;  $m$ , mass of an electron;  $b$ , damping coefficient, i.e., coefficient of friction;  $\eta$  is the coefficient of viscosity.

Below mentioned electron number density satisfies equation (2) as

$$N_e = -\frac{\nu_i - \nu_a}{\alpha} \frac{e^{(\nu_i - \nu_a)t}}{1 - e^{(\nu_i - \nu_a)t}} \quad (4)$$

The energy balance equation [eq. (1)] may be simplified as (Appendix A)

$$\frac{\partial \theta}{\partial \tau} + P(\theta) = Q \quad (5)$$

Expression for electron velocity distribution has been taken from our previous work for this purpose<sup>19</sup> and solution for  $\theta$  is deduced in the integral form as

$$\theta = \frac{Q}{P} - \frac{1}{P} e^{-P\tau} \int \frac{dQ}{d\tau} e^{P\tau} d\tau \quad (6)$$

The medium refractive index has been obtained through the dispersion relation (based on the Appleton Hartree formulation)<sup>20</sup>.

$$n^2 = c^2 \varepsilon_0 \left[ 1 + \frac{2}{2(A - jB) - \frac{D_y^2 + D_z^2}{(A+1) - j(B-C)} \mp \left[ \left\{ \frac{D_y^2 + D_z^2}{(A+1) - j(B-C)} \right\}^2 + 4D_x^2 \right]^{\frac{1}{2}}} \right] \quad (7)$$

Where,

$$A = \frac{\varepsilon_0 m \omega^2}{N_e e^2}, \quad B = \frac{\varepsilon_0 \omega (m v_e + b)}{N_e e^2}, \quad C = \frac{\varepsilon_0 \eta \omega^3 \mu^2}{N_e e^2 c^2}, \quad D_x = \frac{\varepsilon_0 \omega e H_x}{N_e e^2}, \quad D_y = \frac{\varepsilon_0 \omega e H_y}{N_e e^2}, \quad D_z = \frac{\varepsilon_0 \omega e H_z}{N_e e^2}.$$

It is evident that medium refractive index will have two values corresponding to the negative and positive signs involved in the formula. Thus, the electromagnetic wave will split up into two: ordinary and extra-ordinary. As,  $n = \mu - j\chi$ ,  $\mu$  = real part of complex refractive index, and  $\chi$  = absorption coefficient of the medium, where,  $j = \sqrt{-1}$ ,  $\omega$  is the angular frequency of the wave and  $\varepsilon_0$  is the permittivity of free space, eq. (7) is simplified as,

$$(\mu - j\chi)^2 = c^2 \varepsilon_0 \left[ 1 + \frac{(R - jS) \{ 2(A+1) + j2(B-C) \}}{4(A+1)^2 + 4(B-C)^2} \right]$$

With,

$$R = 4A(A+1) - 4B(B-C) - 2D_y^2 \mp D_y^2 - 2D_z^2 \mp D_z^2 \mp 4D_x(A+1) \text{ and}$$

$$S = 2A(B-C) + 4B(A+1) \mp 4D_x(B-C).$$

Explicit expressions for real part of complex refractive index and absorption coefficient of the medium could be found from

$$\mu^2 = \frac{1}{2} c^2 \varepsilon_0 \left\{ \left[ \left\{ 1 + \frac{(A+1)P + (B-C)Q}{2(A+1)^2 + 2(B-C)^2} \right\}^2 + \left\{ \frac{(A+1)Q - (B-C)P}{2(A+1)^2 + 2(B-C)^2} \right\}^2 \right]^{\frac{1}{2}} + \left( 1 + \frac{(A+1)P + (B-C)Q}{2(A+1)^2 + 2(B-C)^2} \right) \right\}$$

$$\chi^2 = \frac{1}{2} c^2 \varepsilon_0 \left\{ \left[ \left\{ 1 + \frac{(A+1)P + (B-C)Q}{2(A+1)^2 + 2(B-C)^2} \right\}^2 + \left\{ \frac{(A+1)Q - (B-C)P}{2(A+1)^2 + 2(B-C)^2} \right\}^2 \right]^{\frac{1}{2}} - \left( 1 + \frac{(A+1)P + (B-C)Q}{2(A+1)^2 + 2(B-C)^2} \right) \right\}$$

Further simplifications are possible if the temperature dependent collision frequency balances the pressure gradient within the medium, then  $\nu_e = -\frac{\nabla P}{m}$ , i.e., the damping coefficient or coefficient of friction effectively vanishes,  $B = \frac{\varepsilon_0 \omega (m \nu_e + b)}{N_e e^2} = 0$ . Also, if medium is non-viscous, then  $\eta = 0$ ,  $C = 0$ , the dispersion relation is simplified as,

$$\mu^2 = \frac{1}{2} c^2 \varepsilon_0 \left[ \{2 + 4A(A+1) - 2D_y^2 - 2D_z^2\} \pm \{D_y^2 + D_z^2 + 4D_x(A+1)\} \right] \text{ and} \quad (8a)$$

$$\chi^2 = 0. \quad (8b)$$

Equation (8b) is simplified for our analysis through

$$\mu^2(A, D_x, D_y, D_z) = \mu^2(A, 0, 0, 0) + D_x F_1(A) + D_y^2 F_2(A) + D_z^2 F_3(A)$$

With

$$\mu^2(A, 0, 0, 0) = c^2 \varepsilon_0 \{1 + 2A(A+1)\}$$

and

$$F_1(A) = c^2 \varepsilon_0 \{1 + 2(A+1)(2A+1)\} \quad (9a)$$

$$F_2(A) = F_3(A) = c^2 \varepsilon_0 \{1 + 2A(A+1)\} \quad (9b)$$

For normal incidence to the ionosphere, condition for vertical reflection of ordinary wave will be  $\mu = 0$  and it leads to  $\omega_0^2 = \frac{N_e e^2}{m \varepsilon_0}$  and normal incidence of extra-ordinary waves to the ionosphere, frequencies splitting are obtained as (Appendix B):

$$\omega_e^2 = \left\{ \omega_0^2 + \frac{1}{2} \left( \frac{e}{m} \right)^2 H^2 \right\} \pm \left[ \left\{ \omega_0^2 + \frac{1}{2} \left( \frac{e}{m} \right)^2 H^2 \right\}^2 - 2N_e^2 e^4 \right]^{\frac{1}{2}} \quad (10)$$

Lastly, the magnitude of electric field has been found as,

$$|E| = \frac{N_e e}{\epsilon_0} \left[ r^2 + \frac{2\mu^2}{c^2 \epsilon_0} (y^2 + z^2) + \frac{3\mu^4}{c^4 \epsilon_0^2} (y^2 + z^2) \right]^{\frac{1}{2}} \quad (11)$$

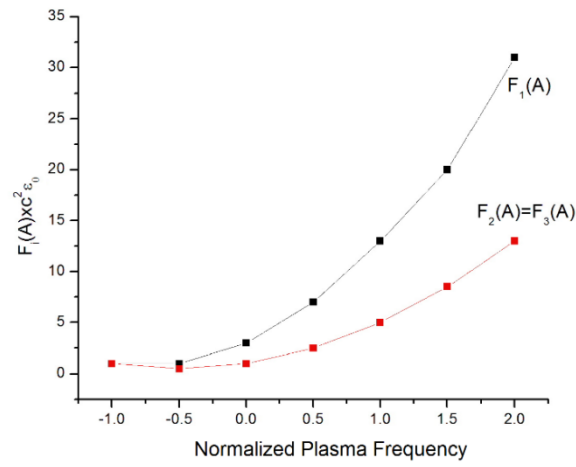
Where,  $r^2 = x^2 + y^2 + z^2$  with  $x, y, z$  are the components of the volume concerned in the rectangular Cartesian co-ordinate system and schematic of which has been shown in Figure 2b.

### 3. RESULTS AND DISCUSSION

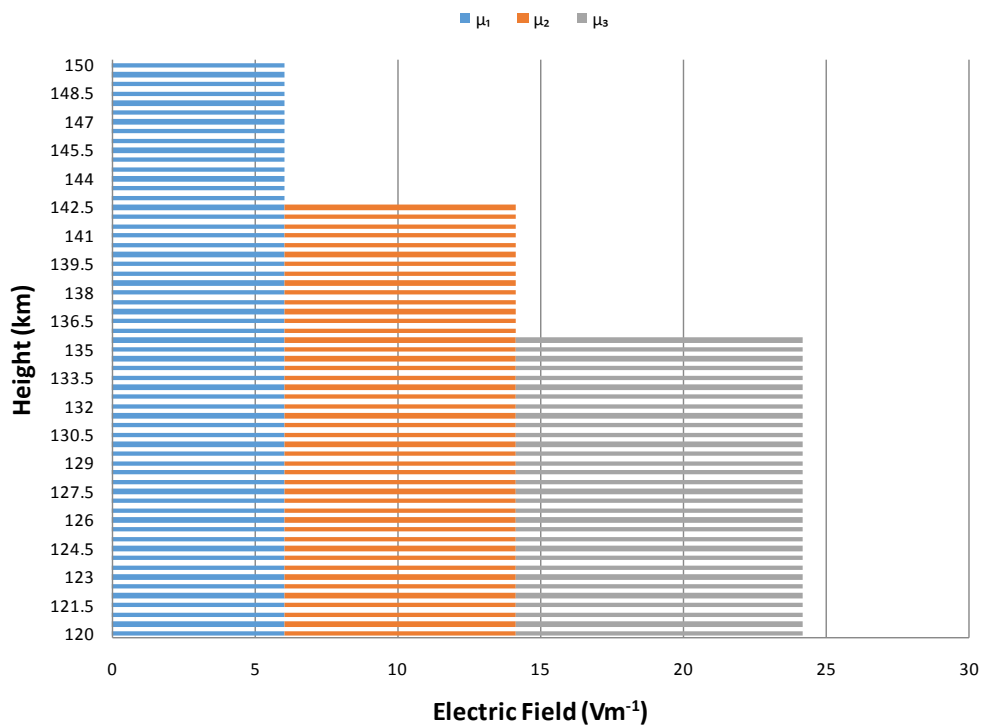
Researchers mainly consider variations of  $\mu^2$  and  $\chi^2$  as functions of plasma frequency for different values of  $\frac{\omega \omega_H}{\omega_0^2}$  and of direction of propagation in magnetoionic theory. Here,  $\omega$  is the angular frequency of the wave,  $\omega_0^2 = \frac{N_e e^2}{m \epsilon_0}$  and  $\omega_H = \frac{eH}{m}$  defined earlier in the text and in literature<sup>21</sup>. In our work, we consider that CME material enters into the medium vertically along the  $z$  direction without angular dependence. If the refractive index is plotted as function of  $\omega_0^2$ , its value changes continuously as plasma frequency increases, corresponding to the continuous change in the nature of wave propagation.

Perturbed zone is chosen having a finite dimension of 450 km along X, 550 km along Y and along Z direction the heights are from 120 to 150 km (Figure 2b). The corresponding components of geomagnetic field are taken respectively as 310 nT, 35500 nT and 22500 nT<sup>22</sup> for the altitude concerned.  $N_e$  is taken as  $\sim 10^9 \text{ m}^{-3}$ ,  $\nu_e(T_e)$  as  $10^3 \text{ Hz}$ ,  $k_p$  as 2.8.<sup>21</sup> It is to be noted that the physical constants are replaced by their standard values in the computation.

The simplified dispersion relation [Eq. (8a)] is analyzed with two functional dependence given in Equations (9a) and (9b). Those variations are shown in Figure 3 (Note: negative values in the abscissa are only for mathematical completeness and without physical significance). This is the condition when temperature dependent collision frequency balances the pressure gradient within the non-viscous medium. But real phenomenon involves greater complexity and requires additional studies along with accurate experimental data for context and constraining theoretical aspects.



**Figure 3:** Functional dependence of refractive index with normalized plasma frequency for non-viscous medium in quasi-dynamic situation



**Figure 4:** Variation of electric field ( $E$ ) with Height for different values of refractive indices ( $\mu$ )

The bar diagram Figure 4 depicts the height variations in the electric field ( $E$ ) for three possible values of the refractive indices ( $\mu$ ) = 0.8, 0.9 and 1.1. These values are likely for the stated physical situation of the medium, obtained from our analyses. Blue ( $\mu_1$ ), orange ( $\mu_2$ ) and grey ( $\mu_3$ ) coloured bars respectively represent the fields for refractive indices  $\mu_1=0.8$ ,  $\mu_2=0.9$  and  $\mu_3=1.1$  for the height ranges from 120 to 150 km as mentioned earlier. It reveals that fields are almost constant throughout a large height range but is sensitive to the refractive indices of the medium.

#### 4. CONCLUSION

Coronal Mass Ejections play a major role in the generation of interplanetary disturbances and can disrupt the magnetic and particle environment near the Earth creating space weather. We have explored the physical situation through the nature of dispersion of the medium by magneto-hydrodynamic formulation. We have established the expressions of complex refractive indices of the medium as well as electric fields within the medium. In computation, we used IRI-2007 data based on VITMO model and MATLAB programs for calculation and Origin 5.0 for plotting. The problem addressed more simplified situations like non-viscous and collision free condition for both ordinary and extra-ordinary waves. We obtain frequency splitting for normal incidence of extra-ordinary waves. Also, we found that the electric fields within the medium remain almost constant throughout a large height range but changes quite with refractive indices of the medium. As the real situation is quite complex thereby demand further studies along with the sophisticated experimental data for the justification of theoretical aspects.

#### ACKNOWLEDGEMENT

Authors gratefully acknowledge the support of the Indian Space Research Organization (ISRO) through S. K. Mitra Centre for Research in Space Environment, Institute of Radio Physics and Electronics, University of Calcutta, Kolkata 700 009, India, in carrying out this work. They wish to thank Mr. P. Hazra to compute few MATLAB coding. They are also thankful to the respected reviewers for their critical comments and valuable suggestions that helped a lot to improve the revised version.

## REFERENCES

1. N. Gopalswamy & B.J. Thompson, *Early life of coronal mass ejections*. J. Atmos. Sol.-Terr. Phys. **62**, (2000) 1457-1469. DOI: [http://doi.org/10.1016/S1364-6826\(00\)00079-1](http://doi.org/10.1016/S1364-6826(00)00079-1).
2. D.F. Webb, *Understanding CMEs and their source regions*. J. Atmos. Sol.-Terr. Phys. **62**, (2000) 1415-1426. DOI: [http://doi.org/10.1016/S1364-6826\(00\)00075-4](http://doi.org/10.1016/S1364-6826(00)00075-4).
3. G. Li, G.P. Zank, & W.K.M. Rice, *Energetic particle acceleration and transport at coronal mass ejection-driven shocks*. J. Geophys. Res. **108**, (2003) 1082. DOI: <http://doi.org/10.1029/2002JA009666>.
4. C. Jacobs & S. Poedts, *Models for coronal mass ejections*. J. Atmos. Sol.-Terr. Phys. **73**, (2011) 1148-1155. DOI: <http://doi.org/10.1016/j.jastp.2010.12.002>.
5. S.S. De, S. Paul, S. Barui, D.K. Haldar & G. Guha, *Earth's atmospheric electricity parameter response during Venus transit*. Terr. Atmos. Ocean. Sci. **26**, (2015) 135-143. DOI: [http://doi.org/10.3319/TAO.2014.10.28.01\(AA\)](http://doi.org/10.3319/TAO.2014.10.28.01(AA)).
6. E. Marsch, *Origin and evolution of the solar wind*, in Solar Activity and its Magnetic Origin, Eds. V. Bothmer & A.A. Hady, Proceedings of the International Astronomical Union, **2** (2006) 259-268. DOI: <http://doi.org/10.1017/S1743921306001980>.
7. D.H. Brooks, I. Ugarte-Urra, & H.P. Warren, *Full-Sun observations for identifying the source of the slow solar wind*. Nature communications **6**, (2015) 5947. DOI: <http://doi.org/10.1038/ncomms6947>.
8. M. Karlický & M. Bárta, *Diagnostics of solar flare reconnection*. Nonlinear Processes Geophys. **11**(4), (2004) 471-483.
9. L. Abbo, L. Ofman, S.K. Antiochos, V.H. Hansteen, L. Harra, Y.K. Ko, G. Lapenta, B. Li, P. Riley, L. Strachan, & R. von Steiger, *Slow Solar Wind: Observations and Modeling*. Space Science Reviews **201**, (2016) 55-108. DOI: <http://doi.org/10.1007/s11214-016-0264-1>.
10. S. Masuda, T. Kosugi, H. Hara, S. Tsuneta & Y. Ogawara, *A loop-top hard X-ray source in a compact solar flare as evidence for magnetic reconnection*. Nature, **371**, (1994) 495-497. DOI: <http://doi.org/10.1038/371495a0>.
11. K. Shibata, *New observational facts about solar flares from Yohkoh studies - Evidence of magnetic reconnection and a unified model of flares*. Adv. Space Res. **17**, (1996) 9-18. DOI: [http://doi.org/10.1016/0273-1177\(95\)00534-L](http://doi.org/10.1016/0273-1177(95)00534-L).
12. S.W. Kahler, & D.V. Reames, *Solar energetic particle production by coronal mass ejection-driven shocks in solar fast-wind regions*. Astrophys. J. **584**, (2003) 1063-1070. DOI: <http://doi.org/10.1086/345780>.
13. E.A. Kasatkina, O.I. Shumilov, M.J. Rycroft, F. Marcz & A.V. Frank-Kamenetsky, *Atmospheric electric field anomalies associated with solar flare/coronal mass ejection events and solar energetic charged particle "Ground Level Events"*. Atmos. Chem. Phys. Discuss. **9**, (2009) 21941-21958.

14. N.A. Murphy, M.P. Miralles, C.L. Pope, J.C. Raymond, H.D. Winter, K.K. Reeves, D.B. Seaton, A.A. van Ballegooijen & J. Lin, *Asymmetric magnetic reconnection in solar flare and coronal mass ejection current sheets*. *Astrophys. J.* **751**, (2012) 1-13.  
DOI: <http://doi.org/10.1088/0004-637X/751/1/56>.
15. D. Bilitza & B.W. Reinisch, *International reference ionosphere 2007: improvements and new parameters*. *Adv. Space Res.* **42**, (2008) 599-609.  
DOI: <http://doi.org/10.1016/j.asr.2007.07.048>.
16. S.S. De, B. Bandyopadhyay, S. Paul, D.K. Haldar & M. Bose, *Heating of the auroral ionosphere by traveling ionospheric disturbances initiated by atmospheric gravity waves*. *Ann. Geophys.* **53**, (2010) 33-37. DOI: <http://doi.org/10.4401/ag-4717>.
17. S. Paul, *Studies on non-linear heating of the auroral region of the ionosphere and some problems related to lower ionosphere*, Ph.D. thesis, Jadavpur University, 2013.  
DOI: <http://doi.org/10.13140/RG.2.1.1239.4080>
18. S.E. El-Khami, R.E. Mcintosh & T. Tang, *Non-linear transient heating of a weakly ionized plasma by an r. f. field*. *J. Appl. Phys.* **41** (1970) 424-427.  
DOI: <http://doi.org/10.1063/1.1658363>.
19. S.S. De, S.K. Adhikari, M. De, B. Bandyopadhyay, A. Guha, S. Paul & B.K. De, *A study on heating of the lower ionosphere during lightning*. *Indian J. Radio Space Phys.* **37**, (2008) 109-113.
20. Satya Prakash, *Electromagnetic Theory and Electrodynamics*. Kedarnath Ramnath & Co. Publishers, Meerut, India (2006).
21. J.A. Ratcliffe, *The Magneto-ionic Theory & its Applications to the Ionosphere*. Cambridge University Press, Cambridge, UK (1962).
22. National Centers for Environmental Information (2016), <http://www.noaa.gov>.

## Appendix A [Derivation of equation (5)]

$$\frac{3}{2} \frac{\partial}{\partial t} (N_e k T_e) + e N_e \vec{v} \cdot (\vec{E} + \vec{v} \times \vec{H}) + G_{eff}(T_e) \nu_e(T_e) - \nabla \cdot \vec{q} - \chi \nabla^2 T_e + Q_i \frac{\partial N_e}{\partial t} = 0$$

Differentiating partially and rearranging the terms,

$$\left( \frac{3}{2} k T_e + Q_i \right) \frac{\partial N_e}{\partial t} + \frac{3}{2} N_e k \frac{\partial T_e}{\partial t} + e N_e \vec{v} \cdot \vec{E} + e N_e \vec{v} \cdot (\vec{v} \times \vec{H}) + G_{eff}(T_e) \nu_e(T_e) - \nabla \cdot \vec{q} - \chi \nabla^2 T_e = 0$$

As  $\vec{v} \cdot (\vec{v} \times \vec{H}) = 0$ , thus

$$\left( \frac{3}{2} k T_e + Q_i \right) \frac{\partial N_e}{\partial t} + \frac{3}{2} N_e k \frac{\partial T_e}{\partial t} + e N_e \vec{v} \cdot \vec{E} + G_{eff}(T_e) \nu_e(T_e) - \nabla \cdot \vec{q} - \chi \nabla^2 T_e = 0$$

Substituting the values of  $\frac{\partial N_e}{\partial t}$  from equation (2)

$$\left(\frac{3}{2}kT_e + Q_i\right)\{(v_i - v_a)N_e - \alpha N_e^2\} + \frac{3}{2}N_e k \frac{\partial T_e}{\partial t} + eN_e \vec{v} \cdot \vec{E} \\ + G_{\text{eff}}(T_e)v_e(T_e) - \nabla \cdot \vec{q} - \chi \nabla^2 T_e = 0$$

Dividing throughout by  $\frac{3}{2}N_e k$  and rearranging,

$$\frac{\partial T_e}{\partial t} + \left(T_e + \frac{2Q_i}{3k}\right)\{(v_i - v_a) - \alpha N_e\} - \frac{2\chi}{3N_e k} \nabla^2 T_e + \frac{2G_{\text{eff}}(T_e)}{3N_e k} v_e(T_e) \\ + \frac{2}{3N_e k} \lambda(T_e) \nabla T_e + \frac{2e}{3k} \vec{v} \cdot \vec{E} = 0$$

Again rearranging the terms to form a partial differential equation of  $T_e$

$$\frac{\partial T_e}{\partial t} + \left[(v_i - v_a) - \alpha N_e - \frac{2\chi}{3N_e k} \nabla^2 + \frac{2}{3N_e k} \lambda(T_e) \nabla\right] T_e \\ = -\frac{2Q_i}{3k} \{(v_i - v_a) - \alpha N_e\} - \frac{2G_{\text{eff}}(T_e)}{3N_e k} v_e(T_e) - \frac{2e}{3k} \vec{v} \cdot \vec{E}$$

Now, let  $\nabla \equiv -jk_p$ , where  $k_p$  be the propagation constant; therefore  $\nabla^2 \equiv -k_p^2$ . Neglecting the imaginary terms, we get

$$\frac{\partial T_e}{\partial t} + \left[(v_i - v_a) - \alpha N_e - \frac{2\chi k_p^2}{3N_e k}\right] T_e = -\frac{2Q_i}{3k} \{(v_i - v_a) - \alpha N_e\} - \frac{2G_{\text{eff}}(T_e)}{3N_e k} v_e(T_e) - \frac{2e}{3k} \vec{v} \cdot \vec{E}$$

$$\text{or, } \frac{\partial}{\partial t} \left(\frac{T_e}{T}\right) + \left[(v_i - v_a) - \alpha N_e - \frac{2\chi k_p^2}{3N_e k}\right] \left(\frac{T_e}{T}\right) \\ = -\frac{2Q_i}{3kT} \{(v_i - v_a) - \alpha N_e\} - \frac{2G_{\text{eff}}(T_e)}{3N_e kT} v_e(T_e) - \frac{2e}{3kT} \vec{v} \cdot \vec{E}$$

$$\text{or, } \frac{\partial}{\partial t} (\theta) + A_1 \theta = A_2 + A_3 + A_4$$

Where

$$A_1 = (v_i - v_a) - \alpha N_e - \frac{2\chi k_p^2}{3N_e k}$$

$$A_2 = -\frac{2Q_i}{3kT} \{(v_i - v_a) - \alpha N_e\}$$

$$A_3 = -\frac{2G_{eff}(T_e)}{3N_e kT} v_e(T_e)$$

$$A_4 = -\frac{2e}{3kT} \vec{v} \cdot \vec{E}$$

Substituting  $v_e(T_e)t = \tau$ , i.e.,  $\frac{\partial}{\partial t} \equiv v_e(T_e) \frac{\partial}{\partial \tau}$  above equation takes the form

$$v_e(T_e) \frac{\partial}{\partial \tau} (\theta) + A_1 \theta = A_2 + A_3 + A_4$$

$$\text{or, } \frac{\partial \theta}{\partial \tau} + \frac{A_1}{v_e(T_e)} \theta = \frac{1}{v_e(T_e)} (A_2 + A_3 + A_4)$$

$$\text{or, } \frac{\partial \theta}{\partial \tau} + P(\theta) = Q \tag{5}$$

With

$$P = \frac{A_1}{v_e(T_e)} = \frac{v_i - v_a}{v_e(T_e)} + \frac{v_i - v_a}{v_e(T_e)} \frac{e^{(v_i - v_a)t}}{1 - e^{(v_i - v_a)t}} - \frac{2\chi k_p^2 \alpha}{3k v_e(T_e) (v_i - v_a)} \frac{e^{(v_i - v_a)t}}{1 - e^{(v_i - v_a)t}}$$

Where explicit expression of  $N_e$  has been used from equation (4) and

$$Q = \frac{1}{v_e(T_e)} (A_2 + A_3 + A_4) = B_2 + B_3 + B_4$$

Where

$$B_2 = -\frac{2Q_i}{3kT v_e(T_e)} \frac{(v_i - v_a)}{1 - e^{(v_i - v_a)t}}$$

$$B_3 = -\frac{2G_{eff}(T_e) \alpha}{3kT (v_i - v_a)} \frac{1 - e^{(v_i - v_a)t}}{e^{(v_i - v_a)t}}$$

$$B_4 = -\frac{2e}{3kTv_e(T_e)} \vec{v} \cdot \vec{E}$$

### Appendix B [Derivation of equation (10)]

For normal incidence of extra-ordinary waves to the ionosphere,  $\mu = 0$  leads to

$$A - \frac{D_y^2 + D_z^2}{2(1+A)} \pm \left[ \frac{(D_y^2 + D_z^2)^2}{4(1+A)^2} + D_x^2 \right]^{\frac{1}{2}} = -1$$

$$\text{or, } (1+A)^2 - \frac{D_y^2 + D_z^2}{2} = \mp \left[ \frac{(D_y^2 + D_z^2)^2}{4} + D_x^2(1+A)^2 \right]^{\frac{1}{2}}$$

Squaring both sides, and after some algebraic simplifications, we get,

$$(1+A)^2[(1+A)^2 - (D_x^2 + D_y^2 + D_z^2)] = 0$$

For extra-ordinary waves,  $(1+A)^2 = D_x^2 + D_y^2 + D_z^2$

$$\left( 1 - \frac{\epsilon_0 m \omega^2}{N_e e^2} \right)^2 = \frac{\epsilon_0^2 \omega^2 H_x^2}{N_e^2 e^2} + \frac{\epsilon_0^2 \omega^2 H_y^2}{N_e^2 e^2} + \frac{\epsilon_0^2 \omega^2 H_z^2}{N_e^2 e^2}$$

$$\text{or, } \left( \frac{N_e e^2 - \epsilon_0 m \omega^2}{N_e e^2} \right)^2 = \frac{\epsilon_0^2 \omega^2}{N_e^2 e^2} (H_x^2 + H_y^2 + H_z^2)$$

$$\text{or, } N_e^2 e^4 + \epsilon_0^2 m^2 \omega^4 - 2mN_e e^2 \omega^2 = \epsilon_0^2 \omega^2 e^2 (H_x^2 + H_y^2 + H_z^2)$$

$$\text{or, } \epsilon_0^2 m^2 \omega^4 - 2mN_e e^2 \omega^2 = \epsilon_0^2 \omega^2 e^2 H^2 - N_e^2 e^4$$

$$\text{or, } \epsilon_0^2 m^2 \omega^4 - (2mN_e e^2 + \epsilon_0^2 e^2 H^2) \omega^2 + N_e^2 e^4 = 0$$

$$\text{or, } \omega^2 = \frac{(2mN_e e^2 + \epsilon_0^2 e^2 H^2) \pm \sqrt{\{(2mN_e e^2 + \epsilon_0^2 e^2 H^2)\}^2 - 4\epsilon_0^2 m^2 N_e^2 e^4}}{2\epsilon_0^2 m^2}$$

$$\text{or, } \omega^2 = \frac{(2mN_e e^2 + \epsilon_0^2 e^2 H^2)}{2\epsilon_0^2 m^2} \pm \left[ \frac{(2mN_e e^2 + \epsilon_0^2 e^2 H^2)^2}{2\epsilon_0^2 m^2} - \frac{4\epsilon_0^2 m^2 N_e^2 e^4}{2\epsilon_0^2 m^2} \right]^{\frac{1}{2}}$$

Substituting  $\omega$  by  $\omega_e$  for extra-ordinary wave

$$\omega_e^2 = \left\{ \omega_0^2 + \frac{1}{2} \left( \frac{e}{m} \right)^2 H^2 \right\} \pm \left[ \left\{ \omega_0^2 + \frac{1}{2} \left( \frac{e}{m} \right)^2 H^2 \right\}^2 - 2N_e^2 e^4 \right]^{\frac{1}{2}} \quad (10)$$