Mathematical patterns hidden in nature

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Do you know that nature is an excellent mathematician?

When we hear the term mathematics do not most of us think of numerals, algebraic equations, geometrical shapes, graphs etc.? But do you know that in nature the patterns in beautiful flowers, shells, fruits and the patterns on the skins and other coverings of animals are all arranged according to certain mathematical principles? Why is the shell of a snail or the shell of a sea snail twisted in a spiral manner? When such organisms grow bigger how do they still retain this shape?

Let us consider this first. Even though it is easy to recognise the patterns in nature' to understand them is not easy. Biologists study the patterns in plants, flowers, fruits and animals and try to give biological explanations and the evolutionary basis for these phenomena.

However most people are unaware or forget that these patterns in living organisms as well as in some non - living things are based on mathematical principles.

Life is involved in a continuous struggle for its survival.

In this struggle the easiest pathway has to be selected to reach its goal while being subjected to the effects of various factors in nature and the environment, and also those exerted by others of its own species around it.

In such situations some of the resulting patterns may be quite complex while others may be quite obvious.

As an example let us consider a simple arithmatic series.

The series 2,4,6,8,10......... is formed by the addition of 2 to the earlier number. It is possible to form such series using some other mathematical method and some of these patterns may be more attractive or fascinating than others.

The following series developed by Fibonacci creates very beautiful patterns.
Here the mathematical method is to begin with 0 and 1 and add the two together to from the next number.
Example:

0+1 = 1
1+1 = 2
1+2 = 3
2+3 = 5
3+5 = 8
5+8 = 13
8+13 = 21
...........and so on.

Then the Fibonacci series of 0,1,1,2,3,5,8,13,21,34 etc. is formed. The next time you visit a flower garden or a home garden with plants bearing flowers count the number of petals in various flowers.

You will notice that most flowers will have either 5 or 8 petals or more.

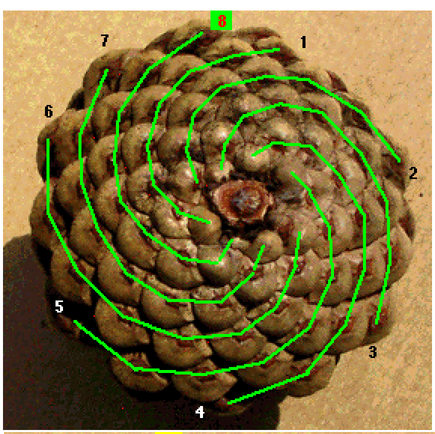
Also you will notice that flowers with four (4) petals are rare. Some flowers have 6 petals and they are mostly composed of 3 pairs of two each. Number 4 does not belong to the Fibonacci series.
If you are living in the hill country or visit a place with a pine tree'

Image 1

Image 2

Image 3

examine a cone of Pinus (Images 4 and 5). Observe carefully the arrangement of their "petals" (sporophylls), and you will see clearly that they are arranged in a spiral manner. When you look more carefully you will be able to see that anti clock wise (turning left) as well as clock wise (turning right)spirals exist. If you count these spiral lines you will see that they will be two consecutive numbers of the Fibonacci series.



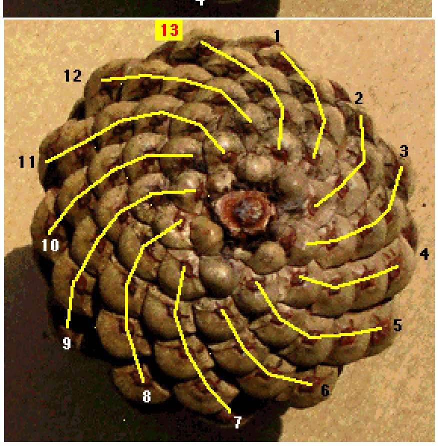


Image 4 and Image 5

Also if you study the distance between the branching points of certain plants you will be able to observe the Fibonacci series.

Consider a young plant after it is newly grown (germinated). It is compulsory for its stem to be strong before it branches out.
Let us assume that it takes two months for this to happen.

If the first branching is after two months and the subsequent branching takes place once a month' then the pattern would be as shown in the figure. A good example for this is the small plant with the scientific name *Achillea Ptarmica*. If you study the environment carefully you will be able to identify a large number of plants with similar patterns.

If we consider the population in a colony of bees you can observe another hidden Fibonacci pattern. We know that a bee colony has a queen, 'worker' females and a few male bees. All female and male bees are the offspring of the queen bee. The female bees are born from the fertilized eggs. Now let us consider the geneology or the family tree of a male bee.

The male bee has only a mother (queen bee). That is only one parent (1). However it has a grand father and a grand mother (The mother and the father of the queen bee) (2). It has no great grand father because its grand father has no father.

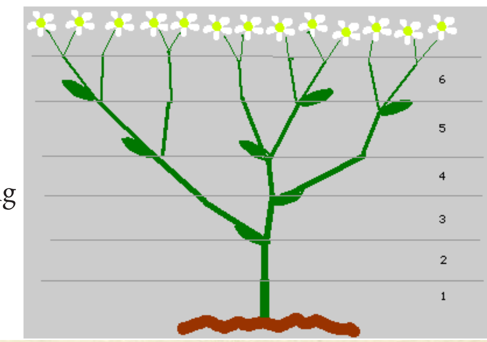


Image 6 and Image 7

It is clear that its series is 1,2,3,5,8,.............

Now let us build a Fibonacci spiral for which we require a graph paper with 13 × 21 squares (Image 8). Now make 1 which is the first number in the (9,6) sector of the graph paper (as shown in the following diagram) Now draw an arc from the top end of the first rectangle to its lower diagonal and extend it to the diagonal of the rectangle on its left. Then extend the arc to the top diagonal of the 2 × 2 rectangle aboue it. Now draw this arc as an unbroken line through to the 3 × 3 rectangle to the 5 × 5 rectangle, 8 × 8 rectangle and finally to the 13 × 13 rectangle. Then you will have a trace as shown below as Image 9. It is a Fibonacci spiral.

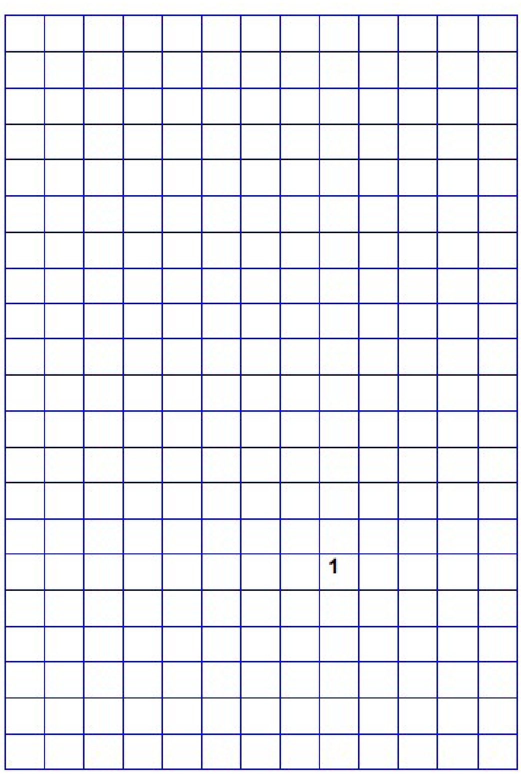


Image 8

Can you see this spiral in the Images 10 to 13.

Now let us observe the number in the Fibonacci series.

1,3,5,8,13,21,34 ....Let us consider the ratio of these numbers with their consecutive number.

2 ÷ 1 = 2
3 ÷ 2 = 1.5
5 ÷ 3 = 1.666...
8 ÷ 5 = 1.6
13 ÷ 8 = 1.625
21 ÷ 13= 1.615
34÷ 25 = 1.619

You will notice that this ratio gradually approaches l.61803. You will notice that the ratio between the number of left handed (anti clock wise) line and the right handed (clock wise) line in the pinus cone is similar.

This is the ‘golden ratio’ of the Fibonacci series.

You have to do a similar thing using the calculator in your computer or the cellular phone.

First enter number 1 and press the button 1/x
Add number 1 and press 1/x again
Add number 1 and press 1/x again

Continue to do this until the number you see on the calculator screen does not change any more. You will see that the number that does not change is 1.61803.

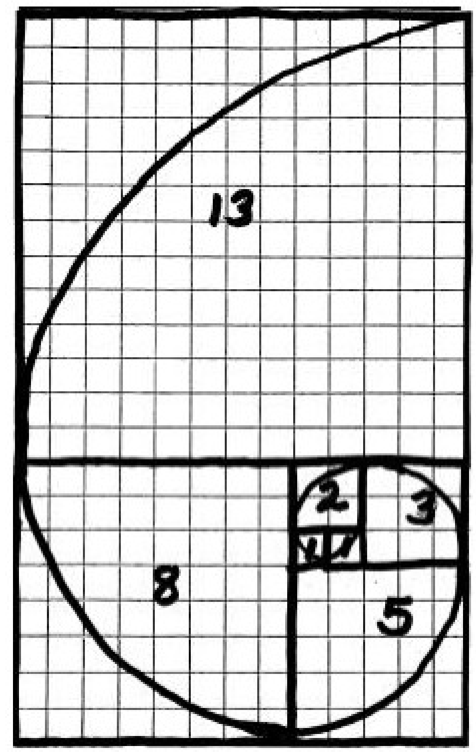


Image 9

How do plants, flowers, fruits or the outer coverings of animals get such a ratio or a series. It is possible to give various scientific answers for this observation. For example if the leaves in a plant are arranged in a spiral manner the upper leaves do not prevent the sunlight reaching the lower leaves. Therefore all leaves receive a certain amount of sunlight and the young leaves receive more sunlight and produce a greater amount of food.

Now let us consider another simple shape that is found in nature. It is no secret that objects in nature starting from atoms, small cells to planets and stars in the universe have a circular or spherical shape. Let us consider a cell first. Let us consider (assume) a two - dimensional cell for convenience. That is, a flat cell on a plane which is a living cell that uses water and food and tries to increase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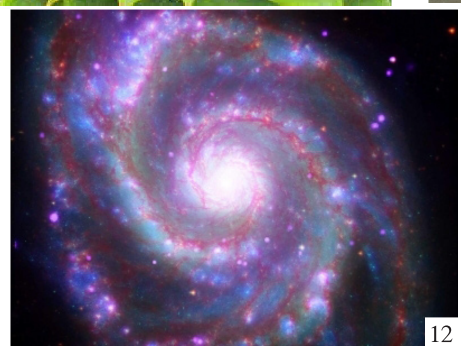
11


12

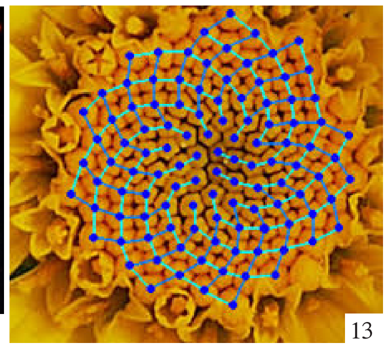

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Image 10, Image 11, Image 12 and Image 13

Table 1:

Shape	Number of sides	Perimeter cm	Area Cm^2	Perimeter/ area
Equilateral triangle	3	45.5901	100	0.456
Square	4	40.0	100	0.4
Hexagon	6	37.22	100	0.372
Circle	Infinite	35.449	100	0.354

Table 2:

Shape	Number of facets	Volume	Surface area
Tetrahedron (Pyramidal)	4	1	7.21
Cube	6	1	6
Octahedron (Double Pyramid)	8	1	5.72
Sphere	Infinite	1	4.84

its size. But a problem faced by the cell is its tendency to lose water though its cell wall. The cell should minimize its perimeter or the length of its cell walls to minimize this loss. In order to achieve this the cell will minimize its perimeter while its area is maximized.

Let us think that the cell has an option of selecting one of the simple shapes of a circle, triangle, square and a hexagon. Let us consider a shape which has an area of $100cm^2$ for comparison.

As can be seen the ratio between the perimeter and the area is minimum in a circle. Now let us consider these dimensional shapes. In this instance the surface area is important rather than the perimeter.

It is clear that the spherical object has the minimum surface area but the maximum volume. The drops of water falling from a height (ex. raindrops), the planets and soap bubbles are spherical because they are trying to reduce their surface area. This is why the soap bubbles you blowout assume only a spherical shape irrespective of the shape of the frame you use.

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