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# A QUANTITATIVE MODEL FOR OPTIMIZATION OF RESOURCE ALLOCATION IN UNIVERSITIES

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## Introduction

In the past decade a gradual decline in the expenditure on university education as a percentage of expenditure on higher education is shown. Further, the expenditure on university education as a percentage of the total expenditure on education over the past few years seem to fluctuate around 15[1]. As a result of these limiting factors there has always been a debate as to how the available resources could be allocated to different universities. It is the administrators of universities who often face the challenge of making the best use of the limited resources available to them. Here the resources may be in the form of academic staff, technical staff, office support staff, funds for purchase of equipment, funds for capital expenditure etc.. More recently, the adoption of quantitative approaches when allocating resources to universities by the University Grants commission (UGC) was witnessed. The UGC established norms for staff-student ratios for different faculties[2] and for the calculation of student loads of different academic programmes offered by the universities,[3] for the purpose of allocating academic and support staff at faculty level. On implementation of these norms the figures calculated by far exceeded the numbers allowed by them. For example, the cadre of permanent teaching staff in universities in 1988 stood at 2042 of which 289 were in excess. Establishment of the norms for staff-student ratios have enabled the UGC to make plans to gradually decrease this excess staff strength. During the plan period (1988-1992), UGC proposes to reduce the excess staff from 289 to 186 by the year 1992. However, particularly at faculty level, the decision makers adopt intuitive or observational approaches that depend on subjective analysis, or they simply repeat someone elses solutions. These methods may not assist in approaching the problem systematically. On the other hand, if a rational approach is adopted the decision makers have a better chance of arriving at a proper decision. In this paper an attempt has been made to use one of the techniques of operations research, namely linear programming, to formulate and solve the resource allocation problem of public universities in Sri Lanka. The linear programming technique is one of the most commercially successful applications of operations research and Wagner[4] reports that there is considerable evidence to suggest that it is ranked highest in economic impact.

Many researchers have also used the linear programming technique to solve allocation problems in educational planning. For example, Adelman[5] developed an educational planning model at national level, Plessner et al.[6] modelled allocation of resources within a university department and Bowles[7] modelled resource allocation in education. Linear programming being a quantitative technique it has been a requirement to quantify the worth of a graduate in all these studies. In Sri Lanka, efforts in the past have been to increase the intake of students to universities as much as possible having been influenced by the social demand for higher education. However, now the emphasis is on labour market rather than the social demand. Therefore, the formulation of the model discussed in this paper is based on the idea that the university is an entrepreneur having the objective of maximizing the worth of the output. The resource allocation problem is conceptualized as a maximizing problem so that the formulation result in the maximization of the relative worth of the output subject to various constraints.

The application of the model is demonstrated through an example. For this purpose the data pertaining to the Faculty of Science at University of Kelaniya was used. The solution of the problem was obtained through the use of a computer software package.[8] The optimal solution to the problem generates student numbers and the resource requirements at department level. The advantages, limitations and further extensions to the model is also discussed.

## DEVELOPMENT OF THE MODEL

### Main Assumptions

i. The measurement of the output from the university is in terms of graduating student numbers.

ii The dropout rate from the university is negligible.

iii. The variation in the results at examinations is attributed to personal qualities of the student and therefore, do not contribute in the calculation of their relative worth.

iv. The students registered in one faculty do not register for courses in other faculties.

### Formulation

The problem is conceptualized as a maximizing problem so that formulation should result in the maximization of the relative worth of the output subject to various constraints. The constraints accommodated in the model represent the resource inputs such as academic staff, supporting staff, and recurrent expenditure and the available physical facilities such as office space, laboratory space, lecture room space and other limiting factors such as academic support services and maintenance of the stability of the institution. The linear programming model developed here is formulated at faculty level.

**Decision Variables**

Due to the nature of the formulation process in optimization models, the decision variables associated with the problem should be those incorporated in the objective function. This condition necessitates the introduction of variables for each category of output from the university. They are as follows;

$x_i$ - number of sudents registered to follow the general degree programme in the faculty offering the 'i' th subject combination.

Usually, the general degree programme consists of a combination of three subjects over a period of three academic years. Further, except the students in the special degree programme, all the others registered at levels such as postgraduate, diploma and certificate follow the courses offered only by the department concerned. This situation leads to the introduction of two more decision variables.

$x_j^k$ - number of students registered in the 'k' th academic programme of department 'j'

$x_j^s$ - number of students registered to follow the subject offered by department 'j' as a subsidiary subject

However, it should be noted that $\Sigma x_j^s$ is a sub-set of $\Sigma x_j^k$ and therefore $x_j^s$ should not appear in the objective function. Instead, they should contribute to the student load calculations at departmental level. The student numbers at the general degree level is defined programme-wise. Therefore, the number of general degree students registered in a particular department will be a function of $x_i$. For simplicity, let $y_j$ be the number of general degree students registered in department 'j'. Then $y_j = f\{x_i\}$. All $x_i$, $x_j^k$ and $x_j^s$ are non-negative variables.

**Objective Function**

It is a maximizing function of the relative worth of the output from all the academic departments within the faculty.

Maximize

$$\left\{\sum_i x_i p_i + \sum_k \sum_j x_j^k p_j^k\right\}$$

where,

$p_i$ - worth of a general degree graduate offering the 'i' th subject combination

$p_j^k$ - worth of a graduate offering the 'k' th academic programme of department 'j'

**Constraints**

The formulation of the constraints associated with academic staff, supporting staff, office space, laboratory space, lecture room space, institutional stability, recurrent expenditure and academic support services is presented in this section.

Academic Staff-

$$\left\{ y_j + x_j^s + \sum_k a_j^k x_j^k \right\} \leq T_j R_j$$

where,

$a_j^k$ - equivalent student load of academic programme 'k' of department 'j' assuming the student load of a general degree student is 1.

$T_j$ - number of staff of department 'j'

$R_j$ - accepted norm for student- teacher ratio applicable for department 'j'

Supporting Staff-

$$\left\{ y_j + x_j^s + \sum_k a_j^k x_j^k \right\} \leq S_j g_j R_j$$

where,

$S_j$ - number of support staff available in department 'j'

$g_j$ - academic staff/support staff ratio applicable to department 'j'

Office Space-

$$\left\{ y_j + x_j + \sum_k a_j^k x_j^k \right\} T_s / R_j + A \leq O_j$$

where,

$A$ - minimum office space required for the Head of department and clerical staff

$T_s$ - minimum office space required per teaching staff member

$O_j$ - total available office space for staff of department 'j'

Laboratory Space-

$$\tfrac{1}{3}\, x_j^k\, w_j^k \le L_j^k$$

where,

$w_j^k$ - minimum lab space required per student of department 'j' following the 'k' th academic programme

$L_j^k$ - total lab space available in department 'j' for the 'k' th academic programme

and $\tfrac{1}{3}\, y_j\, w_j \le L_j$ (General Degree case)

Lecture Room Space-

$M_r \ge$ Max { student strength of each batch of students assigned to the 'r' th lecture room according to the time table }

where,

$M_r$ - capacity of the 'r' th lecture room in student numbers

Institutional Stability -

Sometimes as a result of high costs and other excessive inputs required or due to poor labour market for graduates of certain academic departments, it may happen that in the optimal solution such departments show up as undesirable programmes. However, closing down an existing department may affect the stability of the institution. Therefore, restrictions have to be made to maintain institutional stability.

$$y_j \ge D_j \left\{ t_j \right\}$$

where,

$t_j$ - factor introduced to indicate a lower bound for $y_j$

$D_j$ - present enrolment of general degree students in department 'j'

Recurrent Expenditure-

$$\left\{ e_j \, (y_j + x_j^s) + \sum_k e_j^k x_j^k \right\} \leq V_j$$

where,

- $e_j$ - recurrent expenditure per student of department 'j' offering the general degree programme
- $e_j^k$ - recurrent expenditure per student of department 'j' offering the 'k' th academic programme
- $V_j$ - amount of resources available for recurrent expenditure for department 'j'

Academic Support Services-

Academic support services may be offered by divisions such as the computer unit and the language laboratory.

$$\left\{ b_j \, (y_j + x_j^s) + \sum_k b_j^k x_j^k \right\} \leq C_j$$

where,

- $C_j$ - amount of support service time available for students of department 'j'
- $b_j$ - amount of support service time required for a general degree student of department 'j'
- $b_j^k$ - amount of support service time required for a student of department 'j' offering the 'k' th academic programme

similarly, constraints have to be written for each type of support service used by department 'j'. This constraint may also be formulated at faculty level depending on the feasibility of implementation.

Programme Selection-

In some faculties students who offer certain academic programmes are restricted to specific subject combinations. For example, in the Faculty of Science at University of Kelaniya, those who opt to follow Industrial Management as a subject for the general degree programme have to opt for Mathematics as well. They have to be accommodated separately.

## APPLICATION OF THE MODEL

The model was applied to the situation at the Faculty of Science at University of Kelaniya. The data used in this exercise were pertaining to the year 1985. This time frame was used since at the time of this study, the latest Statistical Handbook of the UGC[9] provided data only upto the year 1985. At that time the faculty had six academic departments and offered eleven academic programmes at general and special degree levels. Only three departments had postgraduate students. These programmes necessitated the introduction of 22 decision variables. A fuller description of the formulation procedure of the linear programming model and the detailed workings of the application is available in an earlier work by the author.[10]

The formulation of the constraints associated with the problem are based on actual figures and therefore, can be formulated fairly close to the real situation. However, the outcome of the solution of the formulated model is heavily dependent on the estimated cost coefficients in the objective function. Determining the worth of graduates who have followed different academic programmes is a very critical issue. Nevertheless it has to be done for the development of the objective function. This sensitive issue has been addressed by many researchers in educational planning. For easiness in application the worth of a graduate is suggested to be considered as the present value of total earnings during his lifetime after the university education less the income he would have earned otherwise and the cost of university education. This is based on the notion that a foregone income is a cost. Estimations based on this criteria involve extensive surveying and therefore, for the purpose of this study the cost coefficients were estimated on subjective judgement. It is worthy of note here that in linear programming analysis, the collection and estimation of relevant data is often the most time consuming part of the project.[11]

In the calculation of staff strength at departmental level, the staff-student ratios given in the Corporate Plan of the UGC of 1987 and the student loads recommended in the Circular Letter of the Planning and Research Division of the UGC was used. But a difficulty in extracting the recurrent expenditure per student by department was experienced due to non availability of classified data. Official records maintained at faculty level also do not have realistic figures on available office space at departmental level. However, obtaining such data may not be difficult if a survey can be administered through high offices in the university administration. In this study the set of constraints dealing with office space was not considered due to practical difficulties encountered in collecting the required information. Further, the Computer Unit of the faculty was at its development stage and did not contribute significantly to warrant its inclusion in the model. In all 45 constraints accommodating, the academic staff, supporting staff, laboratory space, lecture room space, program selection restrictions and institutional stability were used in the model.

**Results**

Due to non availability of reliable estimates for recurrent expenditure per student by departments, initially the optimal solution was obtained without the constraints for recurrent expenditure. The optimal solution obtained along with the actual values of 1985 are presented in Table 1. The figures corresponding to the optimal solution in Table 1 exceed the actual student enrolment in each department. This means that in the Faculty of Science at University of Kelaniya a reasonable increase in student numbers could have been affected in 1985. However when the constraints for recurrent expenditure was included the coresponding optimal solution showed that the actual enrolment at the general degree level, except in the departments of Physics and Industrial Management, were in excess as shown in Table 2. Statistical records[12] indicate that more than 90 percent of the recurrent expenditure in the Faculty of Science is on personal emoluments. Hence, the total recurrent expenditure of the faculty was apportioned on the staff strength of each academic department to arrive at a crude estimate of recurrent expenditure by departments.

**Table 1. Enrolment in the Faculty of Science in 1985 vs Optimal solution obtained without the constraints for recurrent expenditure**

| Department | General Degree Student Numbers | | Special Degree Student Numbers | | M.Sc/M. Phil Student Numbers | |
|---|---|---|---|---|---|---|
| | Optimal | Actual | Optimal | Actual | Optimal | Actual |
| Botany | 80 | 60 | 10 | 10 | 12 | 5 |
| (Bio) | 80 | 60 | | | | |
| Chemistry | | | 10 | 6 | 12 | 2 |
| (Physical) | 66 | 38 | | | | |
| (Pure) | 95 | 63 | | | | |
| Mathematics | | | 10 | 8 | 0 | 0 |
| (Further) | 59 | 42 | | | | |
| Indus. Mgt. | 25 | 19 | 0 | 0 | 0 | 0 |
| Physics | 40 | 39 | 10 | 4 | 0 | 0 |
| Zoology | 80 | 60 | 10 | 10 | 10 | 2 |

There are important features of the linear programming model. For example, the value for surplus and slack variables in the optimal solution indicate the amount of usage of resources and shortfalls. Further, the dual problem generates shadow prices for each input used by the department. By this facility it is possible to decide what particular inputs should be requested if additional resources become available. The shadow price in fact measures the marginal value of the resource concerned.[13]

**Table 2. Enrolment in the Faculty of Science in 1985 as a percentage of the optimal figures obtained with the constraints for recurrent expenditure**

| Department | General Degree Student Numbers Actual as a % of optimal | Special Degree Student Numbers Actual as a % of optimal | M.Sc/M.Phil Student Numbers Actual as a % of optimal |
|---|---|---|---|
| Botany | 130.43 | 100.00 | 41.67 |
| (Bio) | 130.43 | | |
| Chemistry | | 00.00 | 50.00 |
| (Physical) | 292.31 | | |
| (Pure) | 118.87 | | |
| Mathematics | | 80.00 | 00.00 |
| (Further) | 155.56 | | |
| Indus. Mgt. | 76.00 (*) | 00.00 | 00.00 |
| Physics | 97.50 | 40.00 | 00.00 |
| Zoology | 130.43 | 100.00 | 20.00 |

(*) *A percentage less than 100 indicate that the actual figure is less than the optimal number.*

## Conclusions

This paper presents the development of a linear programming model for solving the resource allocation problems in universities. The formulation deals with the situation at faculty level. If it is to be extended to the non-academic divisions too, several norms have to be determined and more constraints have to be developed. There are several advantages of using the linear programming technique. First, by virtue of being a mathematical model it can be subjected to sensitivity analysis. Thus several optimal solutions can be obtained by changing the values of the parameters. In addition, economic interpretation of the model can be done through the associated dual problem. Secondly, the model can be easily solved using the standard computer software packages. Thirdly, the linear programming technique has the capability of handling any type of linear constraints which is adequate to express the resource restrictions close to reality. As in the case of all optimization techniques, once the model is developed it should be validated to see how accurately the model mirrors reality. This can be done experimentally.[14]

This study is intended to assist academic administrators in the resource allocation process by way of providing a quantitative model which would give as the solution the critical areas for investment or withdrawal of resources. However, under no circumstance should the model be viewed as a rigid one. The environmental characteristics which went into the estimation of parameters may change over time. Therefore, the model should be updated to accommodate changes in the parameters.

One of the disadvantages of the present model is the assumption that the students registered in one faculty follow only the courses offered by that faculty. As a result when the course unit system is adopted in the faculties where the students are allowed to follow courses offered in different faculties, the model is not readily applicable. Then new decision variables have to be introduced together with new restrictions.

The results obtained through the solution of the formulated model may be used as a guide and as a measure of control.

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**References**

1. University Grants Commission, (1989), *Statistical Handbook*, Colombo: The University Press.

2. University Grants Commission, (1987), *Corporate Plan for University Education* 1986-1990, Colombo: The University Press.

3. Division of Planning and Research of the UGC, (1987), Letter to all Vice Chancellors and Directors of Institutes, 28th July.

4. Wagner, Harvey M., (1975) *Principles of Management Science*, New Jersey: Prentice-Hall, Inc., p. 29.

5. Adelman, I., (1966), A Linear Programming Model of Educational Planning - A Case Study of Argentina, in the *Theory and Design of Economic Development*, edited by I. Adelman and E. Therbecke, Baltimore: John Hopkins Press, Chap. 14.

6. Plessner, Y., Fox, K.A. and Sanyal, B.C., (1968), On the Allocation of Resources in a University Department, *Metroeconomica*, September-December, pp. 256-271.

7. Bowles, S., (1967), The Efficient Allocation of Resources in Education, *Quarterly Journal of Economics*, May, pp. 189-219.

8. Wickramasooriya, P., (1989), Simplex Software Package, Computer Unit, University of Kelaniya, Kelaniya.

9. Division of Planning and Research of the UGC, (1987), *Statistical Handbook of 1985*, Colombo: The University Press.

10. Galagedera, D. U. A., (1988), Application of Linear Programming Technique to the Resource Allocation Problem of Public Universities in Sri Lanka: A Case Study with Special Reference to University of Kelaniya, A Project Report Prepared for NARESA, No. RG/87/SS/9.

11. Hillier, F.S. and Lieberman, G.J., (1980), *Introduction to Operations Research*, Third Edition, Holden-Day, Inc., p. 41.

12. University of Kelaniya, (1985), Programme Budget-1985, Colombo: The University Press.

13. Cook, T. M. and Russell, R. A., (1981), *Introduction to Management Science*, New Jersey: Prentice-Hall, Inc., p. 41.

14. Thierauf, R.J., Klekamp, R.C. and Ruwe, M.L., (1985), *Management Science*, Ohio: Charles E. Merril Publishing Co., p. 14.