

RESEARCH ARTICLE

Comfort evaluation of bridge based on stochastic process theory

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Submitted: 11 September 2019; Revised: 07 February 2020; Accepted: 08 April 2020


Abstract: Vehicle or pedestrian loads can cause bridge vibration. The pedestrian load is essentially a stochastic process, and the vibration response of pedestrian bridges is also a stochastic process. In order to evaluate the human comfort of a pedestrian bridge comprehensively, this paper presents a comfort evaluation method based on stochastic process theory. The autocorrelation function of the displacement response of a pedestrian bridge under random pedestrian load is derived in this paper. On this basis, the root mean square (RMS) of acceleration is proposed as the comfort evaluation index. Taking an asymmetric single-tower cable-stayed bridge as an example, the modal analysis of the bridge is proceeded by finite element method (FEM). Then the acceleration time histories of four measuring points are obtained by field experiment. FEM results and field experiment results are used to analyse random vibration of the bridge. Through a field experiment and random vibration analysis of the bridge, the acceleration peak value evaluation index and the RMS of acceleration value evaluation index are compared. The results show that the RMS of acceleration index is more comprehensive and reasonable because it considers the whole-time history of the vibration response. The RMS of acceleration index is based on stochastic process theory, so it is more rigorous in mathematical expression and clearer in physical concept. It provides a reference for the evaluation of pedestrian bridge comfort.

Keywords: Acceleration mean square value, comfort, pedestrian bridge, random vibration.

INTRODUCTION

Vehicle or pedestrian loads can cause bridge vibration. If the vibration is high, it can influence the bridge comfort.

In the pursuit of bridge aesthetics, there is a trend in pedestrian bridge design to use a long and flexible structure, which creates the problem of pedestrian-induced vibration. London's Millennium Bridge, Singapore's Changi Mezzanine Bridge, Paris' Solferino Bridge, Japan's T Bridge, and other pedestrian bridges have the problem of excessive human vibration. Since the Millennium Bridge incident in London, scholars have conducted extensive research on the pedestrian-induced vibration of footbridges. Based on the experimental data of the Millennium Bridge in London, Dallard *et al.* (2001) analysed and proposed the interaction model of man-bridge and calculated the critical number of dynamic instability. Živanović *et al.* (2005) reviewed about 200 research studies that dealt with different aspects of the vibration serviceability of footbridges under human-induced load. It has been found that the whole issue is very complex and under-researched. Brownjohn *et al.* (2004) examined real continuous walking forces obtained from an instrumented treadmill and the effect of their random imperfections through a time simulation of the structural response. Chen and Liu (2009) compiled pedestrian footfall observation statistics. Piccardo and Tubino (2012) deduced a spectral model of the modal force induced by stationary groups of pedestrians, considering several sources of randomness. Krenk (2012) developed a frequency representation of vertical pedestrian load and a compact explicit formula for the magnitude of the resulting response. Caprani *et al.* (2012) predicted the vertical acceleration response of a hypothetical footbridge for a sample of

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Table 1: Limit value of footbridge comfort

Standard	Vertical	Transverse
British BS 5400(9352, 2005)	$0.5 f^{0.5} s^{-0.5}$	0.25m/s ²
Germany EN03(RFS2-CT-2007-00033, 2008)	1m/s ²	0.3 m/s ²
Canada OHBDC (Association, 1983)	$0.75 f^{0.75} s^{-0.75}$	--
ISO 3010(ISO, 2017)	0.5—1.0 m/s ²	0.31m/s ² *
European EN 1990(CEN, 2002)	0.7 m/s ²	min(0.14f ^{0.5} , 0.15m/s ²)

* f is natural frequency.

single pedestrians and a crowd of pedestrians using a probabilistic approach. Bassolia *et al.* (2018) evaluated the structural response to vertical pedestrian excitation for a wide range of footbridge and crowd parameters. The current comfort evaluation method mainly takes the acceleration peak as the evaluation index. It is thought that the largest vibration displacement point is the same as the peak acceleration point, so the present comfort evaluation method of a pedestrian bridge is based on the peak acceleration of the whole bridge (Heinemeyer *et al.*, 2009). The limits of peak acceleration in different specifications are listed in Table 1.

However, the pedestrian load is essentially a narrow-band random process (Brownjohn *et al.*, 2004). Therefore, the vibration response of a pedestrian bridge is a stochastic process. The exact values of acceleration and velocity cannot be obtained (Van Nimmen, 2014). The peak acceleration index cannot account for this process, so another evaluation index should be established. In this paper, stochastic process theory was used to analyse the pedestrian-induced vibration of a pedestrian bridge, and a method for evaluating the comfort level of a pedestrian bridge based on the stochastic process is proposed. It is more comprehensive and reasonable, because it considers the whole time-history of the vibration response.

METHODOLOGY

Stochastic process theory for bridge

The most important index to evaluate the vibration response of a pedestrian bridge is the comfort level. The pedestrian load is essentially a stochastic process, and the vibration response of pedestrian bridges is also a stochastic process. Therefore, the comfort level should be evaluated by the stochastic process method.

The pedestrian can only appear at a point in a certain time, so the acceleration means square at the maximal structure response point is representative. According to stochastic process theory, the mean square value is the value of the autocorrelation function if $\tau = t_1 - t_2 = 0$, and it is also the maximum value of the autocorrelation function, that is

$$\sigma_x^2 = E\{X^2(t)\} = R_x(0) = \int_{-\infty}^{\infty} S_x(\omega) d\omega \geq |R_x(\tau)| \quad \dots(1)$$

where R_x is the autocorrelation function. The value of $R_x(0)$ is equal to the total area under the power spectral density curve. S_x is the power spectral density function. The mean square value is the average energy of the stationary stochastic process, which is equal to the average energy of each harmonic component.

The correlation function matrix of the acceleration response can be obtained by the stochastic vibration analysis of a pedestrian bridge. If the load is determined, the acceleration response of the structure is also determined. However, the walking speed of pedestrians has relation to the comfort of the bridge, and the pedestrians will pass through different positions of the bridge in a certain time. The variation of the pedestrian position corresponds to the different correlation function, and the different time course corresponds to a different value of τ .

The dynamic formula of a bridge under pedestrian walking is

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = \sum_{i=1}^N X_i(t) \quad \dots(2)$$

where M is mass matrix of bridge structure, C is damp matrix of bridge structure, K is stiffness matrix of

bridge structure. $u(t)$ is the deformation vector, $\dot{u}(t)$ is the velocity vector, $\ddot{u}(t)$ is the acceleration vector. $X_i(t)$ is single person walking force vector. N is the number of persons. Suppose the stochastic model of the single person walking force is

$$X_i(t) = A \sin(\omega t + \theta) + R \quad \dots(3)$$

where A is the load amplitude, ω is the pedestrian step frequency, θ is the phase of the pedestrian load, which is a random variable uniformly distributed between $-\pi$ and π , and R is the static component of the pedestrian load.

According to stochastic vibration theory, the following equation is used:

$$\sigma^2 \left(\sum_{i=1}^N X_i \right) = \frac{1}{T} \int_0^T \left(\sum_{i=1}^N X_i \right)^2 dt \quad \dots(4)$$

Equation (3) is substituted into Equation (4), then the following equation is used:

$$\sigma^2 \left(\sum_{i=1}^N X_i \right) = N \sigma^2 X_i = N \frac{A^2}{2} \quad \dots(5)$$

If N pedestrians are uniformly distributed on a bridge with length 'L', the $X(x,t)$ standard deviation distributed along the bridge is

$$\sigma [X(x,t)] = \frac{\sqrt{NA}}{\sqrt{2L}} \quad \dots(6)$$

According to the central limit theorem, the sum of N pedestrian loads can be regarded as normal distribution, whose mean value is zero, and the standard deviation is given by the above formula. According to the guarantee rate of 95 %, the force acting evenly along the bridge $X(x,t)$ is

$$X(x,t) = \frac{\sqrt{2NA}}{L} \sin(\omega t + \theta) + NR \quad \dots(7)$$

The static component of the pedestrian load has no effect on the acceleration. According to stochastic vibration theory, the autocorrelation function of the pedestrian load is

$$R_x(\tau) = \frac{A^2}{2} \cos \omega_0 \tau \quad \dots(8)$$

The power spectral density function of the pedestrian loads can be obtained from the Wiener-Khintchine formula (Gadiyar & Padma, 1999):

$$\begin{aligned} S_x(\omega) &= \int_{-\infty}^{\infty} \frac{A^2}{2} \cos \omega_0 \tau \exp(-i\omega \tau) d\tau \\ &= \frac{A^2}{4} \left\{ \int_{-\infty}^{\infty} \exp[i(\omega_0 - \omega)\tau] d\tau + \int_{-\infty}^{\infty} \exp[-i(\omega_0 + \omega)\tau] d\tau \right\} \\ &= \frac{A^2}{4} [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] \quad \dots(9) \end{aligned}$$

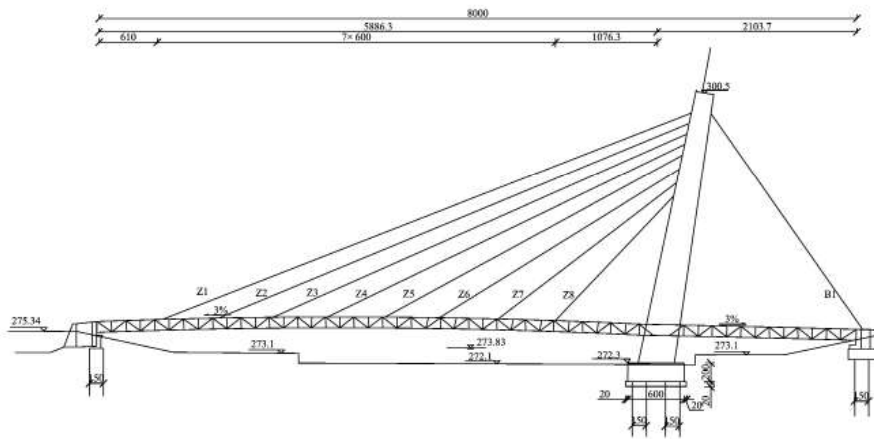
The input power spectrum density is a narrow band random process at the main frequency.

A pedestrian cable-stayed bridge

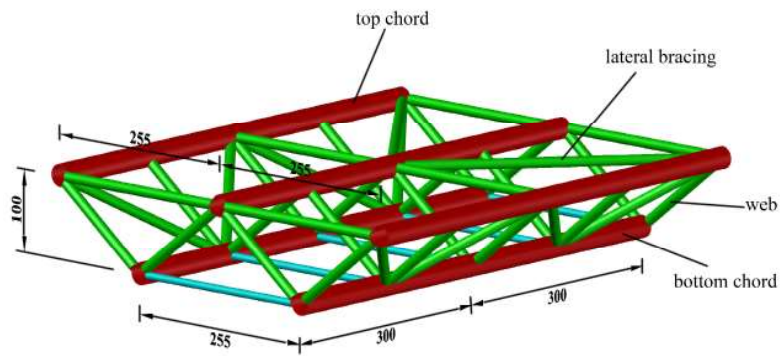
The pedestrian bridge selected is an asymmetric single-tower cable-stayed bridge, as shown in Figure 1. The span of the bridge is $58.7 + 21.3 = 80$ m, and the width is 6 m. The tower is a reinforced concrete arch structure with a solid rectangular cross section. The slope of the tower is 10 degrees to the side span. The tower is 28.2 m high, and the height above the deck is 24.1 m. The girder is a steel tube truss structure. The upper chord and lower chord are set in the shape of a 'W', as shown in Figure 2.



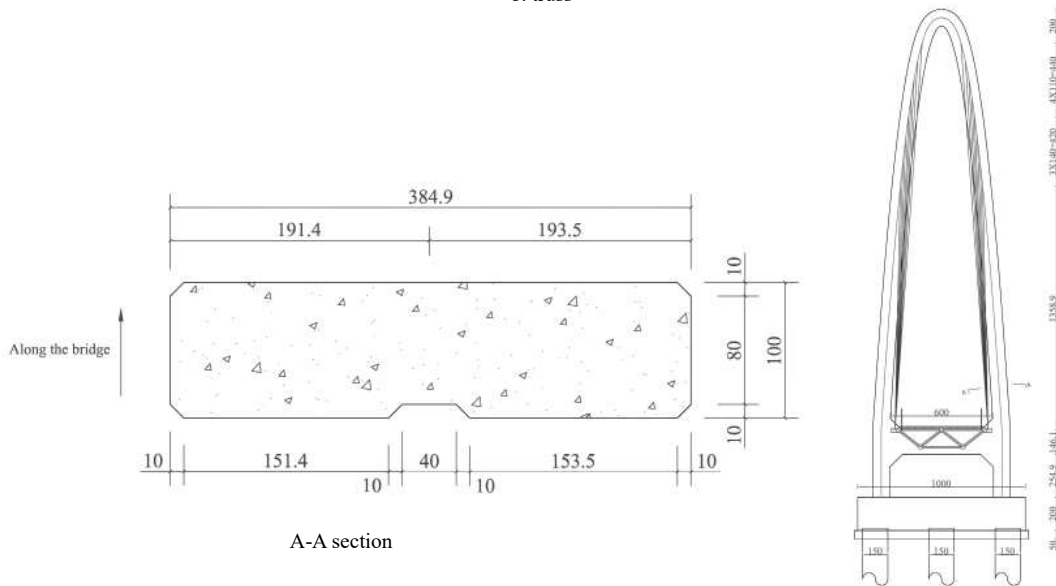
Figure 1: Picture of the pedestrian bridge



a. elevation layout



b. truss



c. tower

Figure 2: General layout of bridge/cm

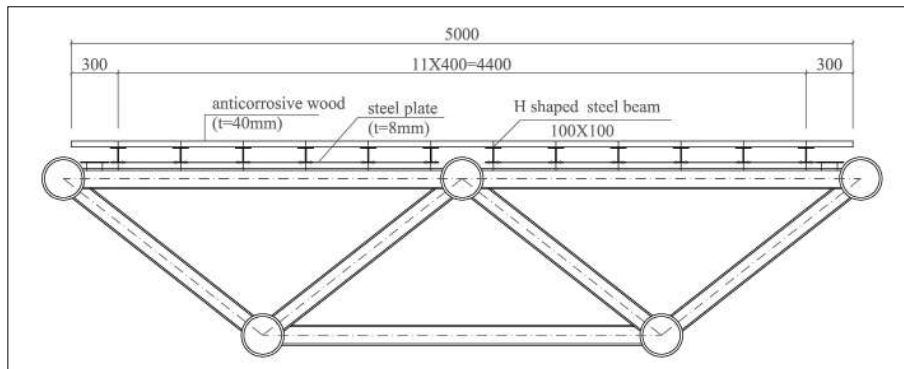


Figure 3: Deck of bridge/mm



Figure 4: TMD under the girder

The chords are made of $\Phi 273 \times 16$ mm steel tube filled with C40 concrete. The webs are made of $\Phi 127 \times 10$ mm steel tube and the lateral bracing are made of $\Phi 95 \times 7$ mm steel tubes. All members are welded. The deck is put on the girder, which is composed of H shape steel beam, steel plate and anticorrosive wood, as shown in Figure 3. There are 8 pairs of spatial cables, which are made of 73 $\Phi 7$ mm high-strength galvanised steel wires with a polyethylene cover. The ultimate tensile strength of the hanger is 1670 MPa. The cable space at the girder is 6m and at the tower is between 1.1m and 1.4m. Both the girder and tower are fixed. The girder is supported at both abutments. Four tuned mass dampers (TMD) are installed at the middle span, as shown in Figure 4. The weight of one TMD is 5KN. The damp of the TMD is 0.05. The natural frequency of the TMD is 1.6 Hz.

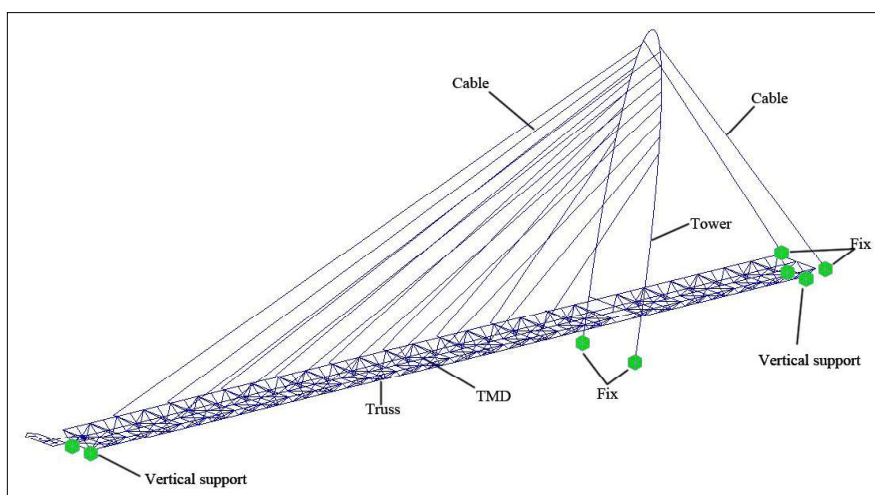


Figure 5: Bridge FEM model diagram

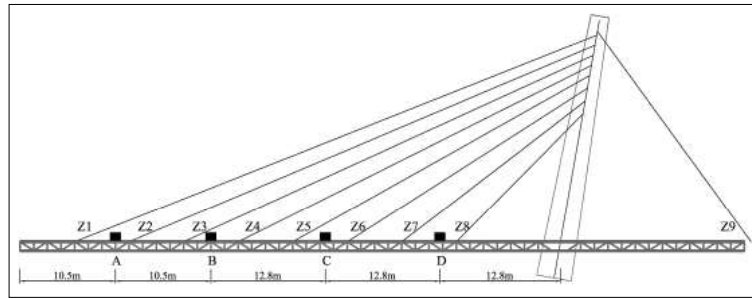


Figure 6: Acceleration sensors layout

The finite element method model

The tower and girder are simulated with 3D beam element. The cable is simulated with link element. However, cable is flexible while link element is straight. The cable sag is depended on tension, so the equivalent module is used for link element, as shown in equation (10). This is Ernst effect.

$$E_{eq} = \frac{E}{1 + \frac{q^2 L^2 A}{12T^3} E} \quad \dots(10)$$

Where E_{eq} is equivalent module of cables, E is module of cables, q is weight of cable per length, A is area of cable and T is tension of cable.

The TMD is simulated with mass element and spring element. The FEM model diagram is shown in Figure 5, and there are 1208 elements.

Modal analysis

Stochastic vibration analysis begins with power spectral density (PSD). Then independent vibration equations should be obtained with the modal analysis method.

Field experiment

Four acceleration sensors A, B, C and D were set up on the top of the deck. They are along the centre line of bridge deck, as shown in Figure 6. The sensitivity of acceleration sensors is $0.3 V/m/s^2$. The measuring range of acceleration sensors is $20 m/s^2$. The pass-band of acceleration sensors is 0.25–80 Hz.

RESULTS AND DISCUSSION

Results

Modal analysis results

The natural frequencies of the bridge were obtained with FEM and are listed in Table 2. The first 20 order modes were found. Only the modes in relation to girder vertical bending are listed, because the vertical motion of the girder was the focus of the study.

Table 2: Natural frequency

Mode order	1	5	6	11	12	16	20
Frequency (Hz)	1.75	3.65	3.99	6.18	7.38	8.90	12.05

The higher order frequencies influence only a little of the vibration of the bridge. Therefore, only the first, fifth, and sixth orders were selected, and their mode shapes can be seen in Figure 7. The first natural frequency is 1.75 Hz, and so the first mode shape is the first-order girder vertical bending. The fifth (3.65 Hz) and sixth (3.99 Hz) natural frequencies are given as the second- and third-order girder vertical bendings, respectively.

Field experiment results

The experiment adopted the artificial excitation method to stimulate the bridge at Point B, and the experimental

load cases are listed in Table 3. In general, excitation methods for bridge dynamic performance test include ambient vibration and external load excitation. The single person jumping at mid-span and a foot formation with 9 persons from bridge head to bridge end were external load excitations. The average weight of each person is 0.7 kN. These external load excitations could excite kinds of structure modes. The structure's natural frequency is 1.75 Hz and the human stride frequency is about 2.0 Hz (Leonard, 1996). Therefore, the excitation frequencies are 1.7Hz and 2.0Hz.

The acceleration time histories of four measuring points were picked up with acceleration sensors. Based on the Nyquist-Shannon sampling theorem (Romo-Cárdenas *et al.*, 2018), the sampling frequency was set as 102.4 Hz and the analytical frequency was set as 51.2 Hz. There were a lot of results; only the measurements of Sensor A in Load Case 2 and Load Case 6 are shown in Figures 8, 9.

Figure 8 shows that the acceleration response centered at 1.7 Hz without the TMD; this was because the load frequency was close to the first-order natural frequency. Figure 9 shows that the acceleration response centered at 3.5 Hz with the TMD; this was because the load frequency was close to the fifth-order natural frequency. This indicates that the structural characteristics changed with the TMD.

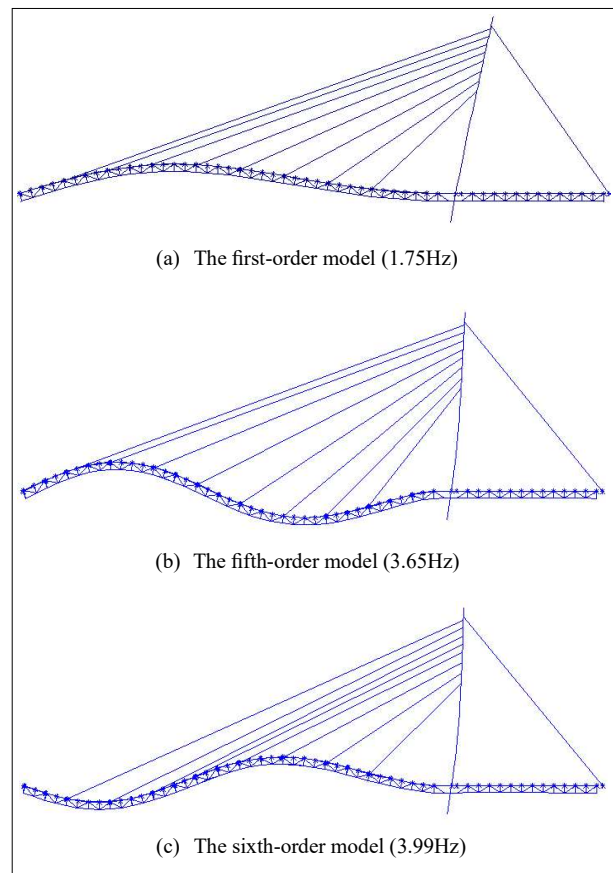
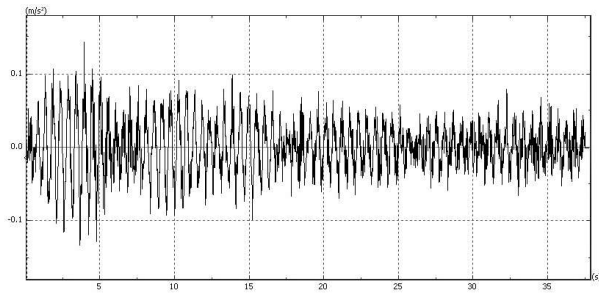


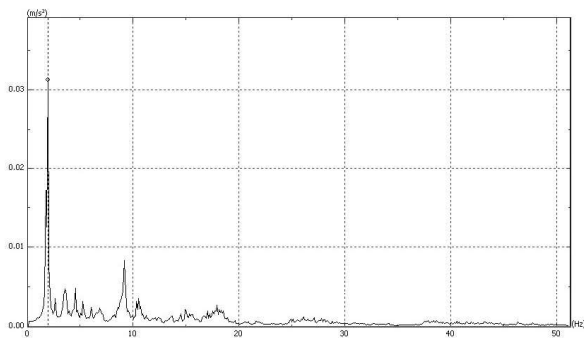
Figure 7: Vibration model of the pedestrian bridge

Table 3: Summary statement of load cases

Load case	Experimental purpose	Excitation mode	Excitation frequency (Hz)
1	Measured natural vibration frequency	Ambient vibration	--
2	Acceleration time history without TMD	A foot formation with 9 persons	1.7
3	Acceleration time history without TMD	A foot formation with 9 persons	2.0
4	Acceleration time history without TMD	Single person jumping at mid-span	1.7
5	Acceleration time history without TMD	Single person jumping at mid-span	2.0
6	Acceleration time history with TMD	A foot formation with 9 persons	1.7
7	Acceleration time history with TMD	A foot formation with 9 persons	2.0
8	Acceleration time history with TMD	Single person jumping at mid-span	1.7
9	Acceleration time history with TMD	Single person jumping at mid-span	2.0

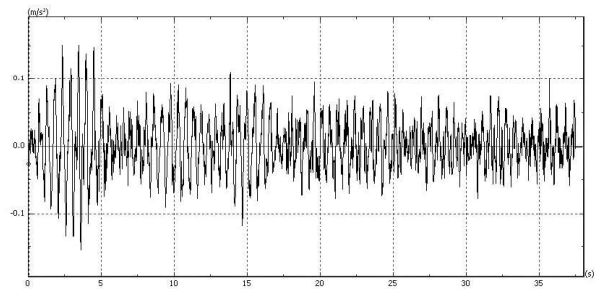


(a) Acceleration time-history curve of sensor A

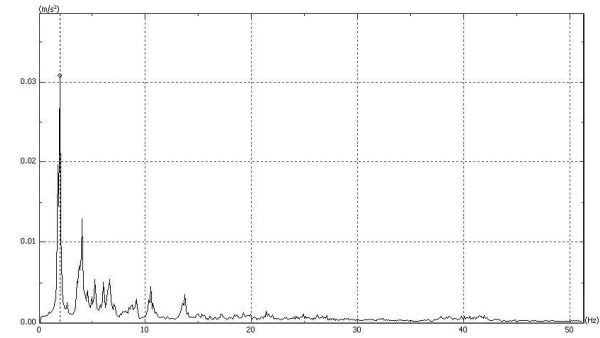


(b) Acceleration spectrum curve of sensor A

Figure 8: Results of load case 2



(a) Acceleration time-history curve of sensor A



(b) Acceleration spectrum curve of sensor A

Figure 9: Results of load case 6

Discussion

RMS of acceleration analysis

In stochastic vibration analysis, displacement, velocity, and acceleration time-history are the only values in the sense of probability. Assuming that the acceleration is subject to normal distribution $N(0, \sigma)$ with a zero mean, the probability density can be calculated inversely according to the correlation function.

The RMS of acceleration summary compared with field measured data is listed in Table 4.

Table 4 shows that the data of the random vibration analysis and the field experiment at Points A, B, and C was similar. This indicates that the random vibration method is a feasible method to analyse the pedestrian-induced vibration of a pedestrian bridge. However, there were errors at Point D. This result is because pedestrian walking is a narrow-band random process, so the vibration

was based mainly on the first-order vibration mode. The response of Point D in the first-order vibration mode was small. In the field experiment, the pedestrian load was not a harmonic load and there were components of other frequencies. Therefore, the excited vibration contained multiple frequencies. This caused errors at Point D.

Vibration reduction rate analysis

The dimension of the RMS of acceleration is the same as acceleration. Hence, RMS of acceleration was compared with peak acceleration and the values are listed in Table 5.

It can be seen from Table 5 that the vibration of the pedestrian bridge was relatively large without the TMD. However, the vibration of the pedestrian bridge was reduced with the TMD. The minimum and maximum peak acceleration vibration reduction rates were 15.69 % and 41.78 %, respectively. The average peak acceleration vibration reduction rates was 28.39 %.

Table 4: RMS of acceleration of measured and analysis

Load case	Measuring point number	Without TMD		With TMD	
		Measured	Random vibration analysis	Measured	Random vibration analysis
Formation with 9 persons 1.7Hz	A	0.025	0.034	0.022	0.028
	B	0.039	0.040	0.034	0.035
	C	0.028	0.033	0.034	0.039
	D	0.036	0.008	0.035	0.009
Formation with 9 persons 2.0Hz	A	0.045	0.060	0.029	0.039
	B	0.081	0.083	0.052	0.053
	C	0.058	0.068	0.038	0.045
	D	0.071	0.022	0.046	0.012
Single person jumping 1.7Hz	A	0.033	0.042	0.023	0.031
	B	0.052	0.053	0.037	0.038
	C	0.035	0.041	0.026	0.030
	D	0.047	0.012	0.034	0.010
Single person jumping 2.0Hz	A	0.125	0.162	0.097	0.126
	B	0.216	0.222	0.167	0.172
	C	0.147	0.170	0.116	0.135
	D	0.193	0.050	0.135	0.042

Table 5: RMS of acceleration and peak acceleration values (m/s²)

Load case	Measuring point number	Peak acceleration			RMS of acceleration		
		Without TMD	With TMD	Vibration reduction rate	Without TMD	With TMD	Vibration reduction rate
Formation with 9 persons 1.7Hz	A	0.1429	0.0866	39.40 %	0.0254	0.022	13.39 %
	B	0.230	0.1339	41.78 %	0.0393	0.0336	14.50 %
	C	0.1547	0.1127	27.15 %	0.0282	0.034	-20.57 %
	D	0.1728	0.1194	30.90 %	0.0355	0.0351	1.13 %
Formation with 9 persons 2.0Hz	A	0.1656	0.1213	26.75 %	0.0452	0.0292	35.40 %
	B	0.2711	0.1881	30.62 %	0.0807	0.0518	35.81 %
	C	0.236	0.1645	30.30 %	0.0581	0.0383	34.08 %
	D	0.2234	0.1639	26.63 %	0.0711	0.0459	35.44 %
Single person jumping 1.7Hz	A	0.1666	0.1126	32.41 %	0.0328	0.0233	28.96 %
	B	0.2027	0.1709	15.69 %	0.0517	0.0366	29.21 %
	C	0.1557	0.115	26.14 %	0.0354	0.0261	26.27 %
	D	0.2087	0.1607	23.00 %	0.0473	0.034	28.12 %
Single person jumping 2.0Hz	A	0.2691	0.2014	25.16 %	0.125	0.0973	22.16 %
	B	0.4115	0.3177	22.79 %	0.2155	0.1673	22.37 %
	C	0.3247	0.2347	27.72 %	0.1471	0.1158	21.28 %
	D	0.3608	0.2605	27.80 %	0.1932	0.1345	30.38 %

The average RMS of acceleration vibration reduction rate was 22.37 %, which is close to the result of the peak acceleration. However, the RMS of acceleration vibration reduction rate at Point C was negative in Load Case 2, which means that the RMS of acceleration response became larger with the TMD. The reasons for the negative vibration efficiency with the TMD are as follows:

(1) A TMD can change the dynamic characteristics of the original structure. Figure 9 (b) shows that the second-order frequency changed into a sensitive frequency with the TMD. Therefore, the RMS of acceleration became larger with the TMD.

(2) The external incentive had a deviation and was not an accurate 1.7 Hz harmonic load.

There is a certain difference between the peak acceleration evaluation results and the RMS of acceleration evaluation results. According to the theory of random vibration, the response of a linear system under a stationary random load will change from a non-stationary process to a stationary process and the mean square value of the transition will gradually increase. The acceleration mean square value is max in a stationary stage, while the instantaneous acceleration value may reach the maximum in a non-stationary stage. Figure 8(b) shows that the acceleration maximum happened in a non-stationary stage before TMD installation. Therefore, it is more reasonable to use the RMS of acceleration as the evaluation index.

CONCLUSIONS

Based on stochastic process theory, the autocorrelation function of the displacement response of a pedestrian bridge under a random pedestrian load is derived in this paper. On this basis, an evaluation method using the mean square value of acceleration at the maximum vibration response of a pedestrian bridge is proposed as the comfort evaluation index. The conclusions of this research are as follows:

(1) The pedestrian-induced vibration response of a pedestrian bridge is a stochastic process. However, the peak acceleration evaluation index ignores the vibration process. Therefore, it is more reasonable to evaluate pedestrian bridges based on stochastic process theory;

(2) With random vibration analysis and a field experiment of a cable-stayed pedestrian bridge, this

research has shown that the RMS of acceleration index based on stochastic process theory is more rigorous in mathematical expression and clearer in physical concept than any other index. It provides a reference for the evaluation of pedestrian bridge comfort.

Acknowledgement

This work was supported by the Foundation of Liaoning Provincial Nature Fund Guidance Projects (2019-ZD-0006, 2019-ZD-0145), the Foundation of Liaoning Provincial Doctoral Scientific Research Projects (20170520138), the Foundation of Liaoning Provincial Department of Education Projects (L2014027).

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