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RESEARCH ARTICLE

On optimal classes of estimators in the presence of some non-sampling errors

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Abstract: During a survey study, an investigator may be incapable of assembling the complete response (i.e., there is non-response) and/or the assembled response is not 100 % true (i.e., measurement errors exist). In this situation, estimation of the population mean under stratified random sampling is not an easy task. Mostly these non-sampling errors, i.e. non-response and measurement error, significantly affect the estimators than sampling errors. To deal with this task, a progressive generalized estimator has been proposed, that can generate a number of estimators based on the availability of conventional and/or non-conventional auxiliary information. Ratio-type, ratio-type exponential, ratio-ratio-type exponential, ratio-product-type exponential, product-type, product-type exponential, product-product-type exponential and product-ratio-type exponential estimators are generated through the proposed generalized estimator. Mathematical properties such as bias, mean squared error and minimum mean squared error of the proposed estimator are derived up to first degree of approximation. The empirical performance of all the estimators in terms of percent relative efficiency is evaluated with the help of a simulation study. It turned out that the proposed estimators outperform when compared with Hansen and Hurwitz (1946) estimator and other competing estimators in this study i.e. Singh and Kumar's (2008), Kumar *et al*. (2015), Azeem and Hanif (2017) and Zahid and Shabbir (2018). It is suggested that the proposed estimators will be applied in case of non-response and measurement errors under stratified random sampling.

Keywords: Auxiliary information, non-sampling errors, percent relative efficiency, stratified random sampling.

INTRODUCTION

In a survey study, a researcher must be concerned about all sources of errors despite a vigilant survey is designed and conducted. These errors may have significant impact on the estimation of population parameters and can be broadly categorized as sampling and non-sampling errors. Sampling error (divergence among the estimate obtained from sample and the true value of the population parameter) has the privileged characteristic of being controllable by the design of sample and the size of sample. Non-sampling errors may be listed as: defective sampling procedures, ambiguity in definitions, faulty measurement techniques, mistakes in recording, incomplete coverage of sample units (non-response) and measurement errors etc. It is important to consider that with the increase of sample size; non-sampling errors increase.

This study emphasises on the estimation of population mean under stratified random sampling in the presence of two major non-sampling errors, i.e. non-response and measurement errors. Non-response refers to a situation when an investigator is incapable of gathering the pertinent information regarding some units of the population. The reasons behind this incapability may be, the respondents are not available at home, the

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respondents are unapproachable, refusal of respondents, etc. Non-response is dominantly seen in surveys based on mails rather than in interviews conducted personally. Maximum information can be obtained by contacting the non-respondents to control this incomplete coverage. Hansen and Hurwitz (1946), Cochran (1977), Rao (1986), Singh and Kumar (2008), Singh *et al*. (2010), Kumar and Chatterjee (2014), Kumar (2015) and others carried out their efforts for the estimation of population parameters under simple random sampling in the presence of non-response.

The difference between recorded value and the true value of a variable is known as measurement error (ME). This concept may be clarified through an example. Let us assume a study associated with a person's age/weight. On investigation, it is possible that the person (respondent) does not remember his/her exact age/weight or may report a rounded figure or may tell a lie in place of his/her exact age/weight. Here the discrepancy among reported and exact age/weight is the measurement error. Cochran (1968; 1977), Shalabh (1997), Singh and Karpe (2009; 2010), Shukla *et al*. (2012) and Sharma and Singh (2013) contributed for the estimation of population mean under the influence of measurement errors.

Continuing the above efforts, Kumar *et al*. (2015), Kumar (2016), Azeem and Hanif (2017) and Irfan *et al*. (2018) suggested new estimators for population mean in simple random sampling. They considered the combined effect of non-response and measurement error. Zahid and Shabbir (2018) suggested a new estimator for the estimation of population mean under stratified random sampling. Their work is based on only conventional auxiliary information.

In context of above scenario, the present study proposed an optimal generalized estimator for the population mean under stratified random sampling without replacement by considering the joint influence of non-response and measurement error. The novelty behind this work is:

(i) Non-conventional auxiliary information as well as conventional auxiliary information is utilized for the first time for progressive estimation under stratified random sampling.

(ii) A variety of estimators can be generated through the proposed generalized estimator: ratio-type, ratio-type exponential, ratio-ratio-type exponential, ratio-product-type exponential, product-type, product-type exponential, product-product-type exponential and product-ratio-type exponential.

Some important assumptions and background

(i) It is assumed that the measurement errors related to study variable and auxiliary variable are purely random;

(ii) Above said errors are uncorrelated with mean zero and variances S_U^2 and S_V^2;

(iii) As per Singh and Karpe (2009), no relationship exists among the true values of the variables and the measurement errors;

(iv) As per Hansen and Hurwitz (1946), the subsample of non-respondents responded when the investigator again contacted them through interview (in case of non-response);

(v) It is also supposed that there is an independence between measurement errors and/or non-responses for both variables under study.

Let us consider a finite population $U = \{U_1, U_2, U_3, ..., U_N\}$ of size N and it can be stratified into L homogenous strata with h^{th} stratum containing $N_h, (h = 1,2,\ldots, L)$ units subject to the restriction that $\sum_{h=1}^{L} N_h = N.$ Assume that each stratum consists of two mutually exclusive groups; $N_h = N_{1h} + N_{2h}$, where N_{1h} are the respondents and N_{2h} are the non-respondents. A sample of size n_h is drawn under simple random sampling without replacement (SRSWOR) from h^{th} stratum such that $\sum_{h=1}^{L} n_h = n$.

Let an investigator email the questionnaires to all n individuals. In h^{th} stratum, he/she receives a response from n_{1h} individuals (i.e., respondents) and n_{2h} individuals does not respond (i.e., non-respondents). The investigator again contacts a subsample $k_h = \frac{n_{2h}}{j_h}; j_h > 1$ of n_{2h} non-respondents. According to assumption (iv), this subsample completely respond when inquiry is done telephonically or through personal interview.

Consider (x_{ih}, y_{ih}) be the pair of values for auxiliary variable X and study variable Y on the ith unit of h^{th} stratum. Let the true values and observed values of the variables (X, Y) represent by (x_{ih}^*, y_{ih}^*) and (X_{ih}^*, Y_{ih}^*) respectively for i^{th} $(i = 1,2, ..., n_h)$ unit in the h^{th} stratum. A complete description of important measures related to study variable and auxiliary variable are given in Table 1.

Table 1: Important measures associated with study variable and auxiliary variable

Measure		Study variable Y	Auxiliary variable X
Measurement error (ME)		$U_{ih}^* = y_{ih}^* - Y_{ih}^*$	$V_{ih}^* = x_{ih}^* - X_{ih}^*$
Sample mean		$\bar{y}_h = {n_h}^{-1} \sum_{i=1}^{n_h} y_{ih}$	$\bar{x}_h = {n_h}^{-1} \sum_{i=1}^{n_h} x_{ih}$
Population	Mean	$\bar{Y}_h = {N_h}^{-1} \sum_{i=1}^{N_h} y_{ih}$	$\bar{X}_h = {N_h}^{-1} \sum_{i=1}^{N_h} x_{ih}$
	Variance	$S_{Yh}^2 = \frac{\sum_{i=1}^{N_h} (Y_{ih} - \bar{Y}_h)^2}{(N_h - 1)}$	$S_{Xh}^2 = \frac{\sum_{i=1}^{N_h} (X_{ih} - \bar{X}_h)^2}{(N_h - 1)}$
	Variance associated with MEs	$S_{Uh}^2 = \frac{\sum_{i=1}^{N_h} (U_{ih} - \bar{U}_h)^2}{(N_h - 1)}$	$S_{Vh}^2 = \frac{\sum_{i=1}^{N_h} (V_{ih} - \bar{V}_h)^2}{(N_h - 1)}$
	Coefficient of Variation	$C_{Yh} = \bar{Y}_h^{-1} S_{yh}$	$C_{Xh} = \bar{X}_h^{-1} S_{xh}$
Non-responding part of population	Variance	$S_{Y(2)h}^2 = \frac{\sum_{i=1}^{N_{2h}} (Y_{ih} - \bar{Y}_h)^2}{(N_{2h} - 1)}$	$S_{X(2)h}^2 = \frac{\sum_{i=1}^{N_{2h}} (X_{ih} - \bar{X}_h)^2}{(N_{2h} - 1)}$
	Variance associated with MEs	$S_{U(2)h}^2 = \frac{\sum_{i=1}^{N_{2h}} (U_{ih} - \bar{U}_h)^2}{(N_{2h} - 1)}$	$S_{V(2)h}^2 = \frac{\sum_{i=1}^{N_{2h}} (V_{ih} - \bar{V}_h)^2}{(N_{2h} - 1)}$
	Coefficient of Variation	$C_{Y(2)h} = \bar{Y}_h^{-1} S_{y(2)h}$	$C_{X(2)h} = \bar{X}_h^{-1} S_{x(2)h}$

Transformation of existing estimators

A review of literature reveals that the estimators for the estimation of population mean under joint effect of MEs and non-response are available only in simple random sampling. The current study has made transformation of all estimators under stratified random sampling.

The traditional Hansen and Hurwitz (1946) estimator along with its variance under stratified random sampling is given by

$$\bar{y}_{st(HH)}^* = \sum_{h=1}^{L} W_h \bar{y}_h^* \quad ...(01)$$

where $\bar{y}_h^* = \left(\frac{n_{1h}}{n_h}\right) \bar{y}_{n_{1h}} + \left(\frac{n_{2h}}{n_h}\right) \bar{y}_{kh}$ and $W_h = \frac{N_h}{N}$

$$Var\left(\bar{y}_{st(HH)}^*\right) = \sum_{h=1}^{L} W_h^2 \left[\lambda_{1h}\left(S_{Yh}^2 + S_{Uh}^2\right) + \lambda_{2h}\left(S_{Y(2)h}^2 + S_{U(2)h}^2\right)\right] \quad ...(02)$$

where $\lambda_{1h} = \frac{(N_h - n_h)}{n_h N_h}$ and $\lambda_{2h} = \frac{N_{2h}(j_h - 1)}{N_h n_h}$

Cochran's (1977) ratio-type estimator may be converted into stratified random sampling as below

$$\bar{y}_{st(CO)} = \sum_{h=1}^{L} W_h \frac{\bar{y}_h^*}{\bar{x}_h^*} \bar{X}_h \quad ...(03)$$

Mean squared error (MSE) of $\bar{y}_{st(CO)}$ is obtained as

$$MSE\left(\bar{y}_{st(CO)}\right) \cong \sum_{h=1}^{L} W_h^2 \Big[\lambda_{1h} \bar{Y}_h^2 \left(C_{Yh}^2 + C_{Xh}^2 - 2\rho_{YXh} C_{Yh} C_{Xh}\right) + \lambda_{2h} \bar{Y}_h^2 \left(C_{Y(2)h}^2 + C_{X(2)h}^2 - 2\rho_{YX(2)h} C_{Y(2)h} C_{X(2)h}\right) + \lambda_{1h} \bar{Y}_h^2 \left(\frac{S_{Uh}^2}{\bar{Y}_h^2} + \frac{S_{Vh}^2}{\bar{X}_h^2}\right) + \lambda_{2h} \bar{Y}_h^2 \left(\frac{S_{U(2)h}^2}{\bar{Y}^2} + \frac{S_{V(2)h}^2}{\bar{X}^2}\right)\Big] \quad ...(04)$$

Another ratio-type estimator was suggested by Rao (1986) in simple random sampling. Given below is the transformation of his estimator and its MSE in stratified random sampling

$$\bar{y}_{st(RA)} = \sum_{h=1}^{L} W_h \frac{\bar{y}_h^*}{\bar{x}_h} \bar{X}_h \quad ...(05)$$

$$MSE(\bar{y}_{st(RA)}) \cong \sum_{h=1}^{L} W_h^2 \Big[\lambda_{1h} \bar{Y}_h^2 (C_{Yh}^2 + C_{Xh}^2 - 2\rho_{YXh} C_{Yh} C_{Xh}) + \lambda_{2h} S_{Y(2)h}^2 + \lambda_{1h} \bar{Y}_h^2 \left(\frac{S_{Uh}^2}{\bar{Y}_h^2} + \frac{S_{Vh}^2}{\bar{X}_h^2} \right) + \lambda_{2h} S_{U(2)h}^2 \Big] \quad ...(06)$$

Singh and Kumar's (2008) simple estimator is transformed into stratified random sampling in this way

$$\bar{y}_{st(SK)} = \sum_{h=1}^{L} W_h \bar{y}_h^* \left(\frac{\bar{X}_h}{\bar{x}_h^*} \right) \left(\frac{\bar{X}_h}{\bar{x}_h} \right) \quad ...(07)$$

MSE of $\bar{y}_{st(SK)}$ is given by

$$MSE(\bar{y}_{st(SK)}) \cong \sum_{h=1}^{L} W_h^2 [\lambda_{1h} \bar{Y}_h^2 (C_{Yh}^2 + 4C_{Xh}^2 - 4\rho_{YXh} C_{Yh} C_{Xh}) + \lambda_{2h} \bar{Y}_h^2 (C_{Y(2)h}^2 + C_{X(2)h}^2 - 2\rho_{YX(2)h} C_{Y(2)h} C_{X(2)h}) + \lambda_{1h} \bar{Y}_h^2 \left(\frac{S_{Uh}^2}{\bar{Y}_h^2} + 4\frac{S_{Vh}^2}{\bar{X}_h^2} \right) + \lambda_{2h} \bar{Y}_h^2 \left(\frac{S_{U(2)h}^2}{\bar{Y}_h^2} + \frac{S_{V(2)h}^2}{\bar{X}_h^2} \right)] \quad ...(08)$$

Kumar *et al*. (2015) suggested an exponential ratio type estimator in simple random sampling. Its conversion into stratified random sampling is given by

$$\bar{y}_{st(KU)} = \sum_{h=1}^{L} W_h \bar{y}_h^* \left(\frac{\acute{\bar{x}}_h^{**}}{\bar{X}_h} \right) \left[\alpha_h \exp\left(\frac{\bar{x}_h^* - \acute{\bar{x}}_h^*}{\bar{x}_h^* + \acute{\bar{x}}_h^*} \right) (1-\alpha_h) \exp\left(\frac{\acute{\bar{x}}_h^* - \bar{x}_h^*}{\acute{\bar{x}}_h^* + \bar{x}_h^*} \right) \right] \quad ...(09)$$

where $\acute{\bar{x}}_h^{**} = \frac{N_h \bar{X}_h - n_h \bar{x}_h}{N_h - n_h}, \acute{\bar{x}}_h^* = \frac{N_h \bar{X}_h - n_h \bar{x}_h^*}{N_h - n_h}$

and α_h is the weight chosen in a way that minimizes the MSE. The weight of α_h and resulting MSE is determined as

$$\alpha_h = \left(\frac{N_h + 2n_h}{2N_h} \right) - \left(\frac{N_h - n_h}{N_h} \right) \left(\frac{\bar{X}_h}{\bar{Y}_h} \right) \left(\frac{\lambda_{1h} \rho_{YXh} S_{Yh} S_{Xh} + \lambda_{2h} \rho_{YX(2)h} S_{Y(2)h} S_{X(2)h}}{\lambda_{1h} (S_{Xh}^2 + S_{Vh}^2) + \lambda_{2h} (S_{X(2)h}^2 + S_{V(2)h}^2)} \right)$$

$$MSE_{min}(\bar{y}_{st(KU)}) \cong \sum_{h=1}^{L} W_h^2 \Big[\lambda_{1h} (S_{Yh}^2 + S_{Uh}^2) + \lambda_{2h} (S_{Y(2)h}^2 + S_{U(2)h}^2) - \frac{(\lambda_{1h} \rho_{YXh} S_{Yh} S_{Xh} + \lambda_{2h} \rho_{YX(2)h} S_{Y(2)h} S_{X(2)h})^2}{\lambda_{1h} (S_{Xh}^2 + S_{Vh}^2) + \lambda_{2h} (S_{X(2)h}^2 + S_{V(2)h}^2)} \Big] \quad ...(10)$$

Azeem and Hanif (2017) proposed three estimators in simple random sampling. Given below are their modified versions in stratified random sampling

$$\bar{y}_{st(AH1)} = \sum_{h=1}^{L} W_h \bar{y}_h^* \left(\frac{\acute{\bar{x}}_h^*}{\bar{X}_h} \right) \left(\frac{\acute{\bar{x}}_h^*}{\bar{x}_h^*} \right) \quad ...(11)$$

$$\bar{y}_{st(AH2)} = \sum_{h=1}^{L} W_h \bar{y}_h^* \left(\frac{\acute{\bar{x}}_h^*}{\bar{X}_h} \right) \exp\left(\frac{\acute{\bar{x}}_h^* - \bar{x}_h^*}{\acute{\bar{x}}_h^* + \bar{x}_h^*} \right) \quad ...(12)$$

$$\bar{y}_{st(AH3)} = \sum_{h=1}^{L} W_h \bar{y}_h^* \left[\alpha_h \exp\left(\frac{\bar{X}_h - \bar{x}_h^*}{\bar{X}_h + \bar{x}_h^*} \right) + (1-\alpha_h) \exp\left(\frac{\acute{\bar{x}}_h^* - \bar{x}_h^*}{\acute{\bar{x}}_h^* + \bar{x}_h^*} \right) \right] \quad ...(13)$$

where α_h is the same as stated above.

Following expression summarizes the MSEs of Azeem and Hanif's (2017) three estimators

$$MSE(\bar{y}_{st(AHi)}) \cong \sum_{h=1}^{L} W_h^2 \Big[\lambda_{1h} \bar{Y}_h^2 (C_{Yh}^2 + \mu_{ih}^2 C_{Xh}^2 - 2\mu_{ih} \rho_{YXh} C_{Yh} C_{Xh}) + \lambda_{2h} \bar{Y}_h^2 (C_{Y(2)h}^2 + \mu_{ih}^2 C_{X(2)h}^2 - 2\mu_{ih} \rho_{YX(2)h} C_{Y(2)h} C_{X(2)h}) + \lambda_{1h} \bar{Y}_h^2 \left(\frac{S_{Uh}^2}{\bar{Y}_h^2} + \mu_{ih}^2 \frac{S_{Vh}^2}{\bar{X}_h^2} \right) + \lambda_{2h} \bar{Y}_h^2 \left(\frac{S_{U(2)h}^2}{\bar{Y}_h^2} + \mu_{ih}^2 \frac{S_{V(2)h}^2}{\bar{X}_h^2} \right) \Big] \text{ for } i = 1,2,3 \quad ...(14)$$

where $\mu_{1h} = \frac{N_h + n_h}{N_h - n_h}, \ \mu_{2h} = \frac{1}{2} \left(\frac{N_h + 2n_h}{N_h - n_h} \right)$

and $\mu_{3h\,(opt)} = \dfrac{[\lambda_{1h}\rho_{YXh}C_{Yh}C_{Xh} + \lambda_{2h}\rho_{YX(2)h}C_{Y(2)h}C_{X(2)h}]}{\lambda_{1h}\left(\dfrac{S_{Xh}^2 + S_{Vh}^2}{\bar{X}_h^2}\right) + \lambda_{2h}\left(\dfrac{S_{X(2)h}^2 + S_{V(2)h}^2}{\bar{X}_h^2}\right)}$

Recently, Zahid and Shabbir (2018) proposed an estimator in stratified random sampling detailed below

$$\bar{y}_{st(ZS)} = \sum_{h=1}^{L} W_h \left[\{m_{1h}\bar{y}_h^* + m_{2h}(\bar{X}_h - \acute{\bar{x}}_h^*)\}\left(\frac{\bar{X}_h}{\acute{\bar{x}}_h^*}\right)^{\gamma_h} \exp(1-\gamma_h)\left(\frac{\bar{X}_h - \acute{\bar{x}}_h^*}{\bar{X}_h + \acute{\bar{x}}_h^*}\right)\right] \quad ...(15)$$

where m_{1h} and m_{2h} are the suitably chosen weights and γ_h is the scalar, and takes the values $(-1,0,1)$.

Minimum MSE of $\bar{y}_{st(ZS)}$ is given as

$$MSE_{min}(\bar{y}_{st(ZS)}) \cong \sum_{h=1}^{L} W_h^2\left[\bar{Y}_h^2 - \frac{(M_{1h}M_{5h}^2 + M_{2h}M_{4h}^2 - 2M_{3h}M_{4h}M_{5h})}{(M_{1h}M_{2h} - M_{3h}^2)}\right] \quad ...(16)$$

where

$M_{1h} = \bar{Y}_h^2 + q_{2h} + e_h^2 t_h^2 R_h^2 q_{1h} + 4e_h t_h R_h q_{3h} + 2f_h t_h^2 R_h^2 q_{1h}$,

$M_{2h} = t_h^2 q_{1h}$

$M_{3h} = t_h q_{3h} + 2e_h t_h^2 R_h q_{1h}$,

$M_{4h} = \bar{Y}_h^2 + e_h t_h R_h q_{3h} + f_h t_h^2 R_h^2 q_{1h}$ and

$M_{5h} = e_h t_h^2 R_h q_{1h}$

here

$e_h = \dfrac{1+\gamma_h}{2}$, $t_h = \dfrac{n_h}{N_h - n_h}$, $R_h = \dfrac{\bar{Y}_h}{\bar{X}_h}$ and

$f_h = \dfrac{\gamma_h^2 + 4\gamma_h + 3}{8}$

METHODOLOGY

This section suggests a new generalized estimation for the population mean in stratified random sampling without replacement under the joint influence of measurement errors and non-response. The following generalized estimator is suggested.

$$T_{st} = \sum_{h=1}^{L} W_h\left[\omega_{1h}\bar{y}_h^*\left(\frac{\bar{Z}_h}{\bar{z}_h}\right)^{g_h} + \omega_{2h}\bar{y}_h^*\exp\left(\frac{\delta_h(\bar{Z}_h - \bar{z}_h)}{\bar{Z}_h + \bar{z}_h}\right)\right] \quad ...(17)$$

where $\bar{Z}_h = a_h\bar{X}_h + b_h$ and $\bar{z}_h = a_h\acute{\bar{x}}_h^* + b_h$

and $a_h(\neq 0), b_h$ are either available conventional or non-conventional auxiliary information or any real number. $(\omega_{1h}, \omega_{2h})$ are the suitably chosen weights that cause the MSE minimum. (g_h, δ_h) being constants assume values (-1, 0, 1). These constants help us to generate a variety of estimators given in Table 2 (See Appendix).

Remark 1: Conventional parameters of the h^{th} stratum may be listed as median Q_{2h}, standard deviation S_{Xh}, coefficient of variation C_{Xh}, coefficient of skewness $\beta_{1h}(X)$, coefficient of kurtosis $\beta_{2h}(X)$ and coefficient of correlation ρ_{YXh} etc.

Remark 2: Non-conventional parameters of the h^{th} stratum include quartile deviation QD_h, mid-range MR_h, trimmed mean TM_h, Hodge-Lehmann estimator HL_h, etc. where

$QD_h = \dfrac{Q_{3h} - Q_{1h}}{2}$,

$MR_h = \dfrac{X_{(1)h} + X_{(N)h}}{2}$,

$TM_h = \frac{Q_{1h} + 2Q_{2h} + Q_{3h}}{4}$ and

$HL_h = \text{median}\left(\frac{X_{jh} + X_{kh}}{2}\right), 1 \le jh \le kh \le N$

Remark 3: We can generate a number of estimators by substituting known parameters (given in remark 1 and remark 2) in a_h and b_h in $T_{sti}^{(1)}$. Similar process can be adopted for $T_{sti}^{(2)}, T_{sti}^{(3)}, \ldots, T_{sti}^{(8)}$ to gain many more estimators. Some of them ($T_{sti}^{(1)}$ and $T_{sti}^{(3)}$) are presented in Table 3 (See Appendix).

It is necessary to describe some expressions to obtain the mathematical properties of the proposed estimator T_{st}. These properties include bias, MSE and minimum MSE.

$\omega_{Yh}^* = \sum_{i=1}^{n_h}(Y_{ih}^* - \bar{Y}_h)$, $\omega_{Xh}^* = \sum_{i=1}^{n_h}(X_{ih}^* - \bar{X}_h)$,

$\omega_{Uh}^* = \sum_{i=1}^{n_h} U_{ih}^*$ and $\omega_{Vh}^* = \sum_{i=1}^{n_h} V_{ih}^*$

Adding ω_{Yh}^* and ω_{Uh}^* dividing by n_h, we get

$$\frac{1}{n_h}(\omega^*_{Yh} + \omega^*_{Uh}) = \frac{1}{n_h}\sum_{i=1}^{n_h} y^*_{ih} - \overline{Y}_h$$

$$\bar{y}^*_h = \overline{Y}_h + \frac{1}{n_h}(\omega^*_{Yh} + \omega^*_{Uh}) \qquad ...(18)$$

In the same way, we have

$$\bar{x}^*_h = \overline{X}_h + \frac{1}{n_h}(\omega^*_{Xh} + \omega^*_{Vh}) \qquad ...(19)$$

After taking expectations, we have

$$\left.\begin{aligned} E\left(\frac{\omega^*_{Xh} + \omega^*_{Vh}}{n_h}\right)^2 &= \lambda_{1h}\left(S^2_{Xh} + S^2_{Vh}\right) + \lambda_{2h}\left(S^2_{X(2)h} + S^2_{V(2)h}\right) = q_{1h}(\text{Say}) \\ E\left(\frac{\omega^*_{Yh} + \omega^*_{Uh}}{n_h}\right)^2 &= \lambda_{1h}\left(S^2_{Yh} + S^2_{Uh}\right) + \lambda_{2h}\left(S^2_{Y(2)h} + S^2_{U(2)h}\right) = q_{2h}(\text{Say}) \\ E\left[\left(\frac{\omega^*_{Yh} + \omega^*_{Uh}}{n_h}\right)\left(\frac{\omega^*_{Xh} + \omega^*_{Vh}}{n_h}\right)\right] &= \lambda_{1h}\rho_{YXh}S_{Yh}S_{Xh} + \lambda_{2h}\rho_{YX(2)h}S_{Y(2)h}S_{X(2)h} = q_{3h}(\text{Say}) \end{aligned}\right\} \qquad ...(20)$$

The new generalized estimator given in equation (17) can be rewritten using equation (18) and equation (19) in the following way. Expression is taken up to first degree of approximation.

$$T_{st} - \overline{Y} \cong \sum_{h=1}^{L} W_h\Big[\overline{Y}_h(\omega_{1h} + \omega_{2h} - 1) + (\omega_{1h} + \omega_{2h})\left(\tfrac{\omega^*_{Yh}+\omega^*_{Uh}}{n_h}\right) + \overline{Y}_h\tau_h\left(\omega_{1h}g_h n_h + \tfrac{1}{2}\omega_{2h}\delta_h\right)\left(\tfrac{\omega^*_{Xh}+\omega^*_{Vh}}{n_h}\right)$$

$$+ \overline{Y}_h\tau^2_h\left(\tfrac{1}{4}\omega_{2h}\delta_h\left(1 + \tfrac{\delta_h}{2}\right) - \tfrac{1}{2}\omega_{1h}n^2_h g_h(g_h + 1)\right)\left(\tfrac{\omega^*_{Xh}+\omega^*_{Vh}}{n_h}\right)^2 + \tau_h\left(\omega_{1h}g_h n_h + \tfrac{1}{2}\omega_{2h}\delta_h\right)\left(\tfrac{\omega^*_{Yh}+\omega^*_{Uh}}{n_h}\right)\left(\tfrac{\omega^*_{Xh}+\omega^*_{Vh}}{n_h}\right)\Big] \qquad ...(21)$$

where $\tau_h = \dfrac{a_h}{(N_h - n_h)(a_h\overline{X}_h + b_h)}$

The bias of T_{st} is obtained up to first degree of approximation in this way

$$Bias\ (T_{st}) = E(T_{st} - \overline{Y})$$

$$Bias\ (T_{st}) \cong \sum_{h=1}^{L} W_h\left[\overline{Y}_h(\omega_{1h} + \omega_{2h} - 1) + \overline{Y}_h\tau^2_h q_{1h}\left(\frac{1}{4}\omega_{2h}\delta_h\left(1 + \frac{\delta_h}{2}\right) - \frac{1}{2}\omega_{1h}n^2_h g_h(g_h + 1)\right) + \tau_h q_{3h}\left(\omega_{1h}g_h n_h + \frac{1}{2}\omega_{2h}\delta_h\right)\right] \qquad ...(22)$$

The MSE of the new estimator T_{st} can be obtained by squaring both sides of equation (21) and taking the expectation

$$MSE\ (T_{st}) \cong \sum_{h=1}^{L} W^2_h\left[\overline{Y}_h + \omega^2_{1h}A_h + \omega^2_{2h}B_h - \omega_{1h}C_h - \omega_{2h}D_h + \omega_{1h}\omega_{2h}E_h\right]$$

where

$$A_h = \left[\overline{Y}_h - q_{1h}\overline{Y}^2_h\tau^2_h n^2_h g_h + q_{2h} + 4q_{3h}\overline{Y}_h\tau_h n_h g_h\right]$$

$$B_h = \left[\overline{Y}_h + \frac{1}{2}q_{1h}\overline{Y}^2_h\tau^2_h\delta_h(1 + \delta_h) + q_{2h} + 2q_{3h}\overline{Y}_h\tau_h\delta_h\right]$$

$$C_h = \left[2\overline{Y}_h - q_{1h}\overline{Y}^2_h\tau^2_h n^2_h g_h(1 + g_h) + 2q_{3h}\overline{Y}_h\tau_h n_h g_h\right]$$

$$D_h = \left[2\overline{Y}_h + \frac{1}{2}q_{1h}\overline{Y}^2_h\tau^2_h\delta_h\left(1 + \frac{\delta_h}{2}\right) + q_{3h}\overline{Y}_h\tau_h\delta_h\right]$$

$$E_h = \left[2\overline{Y}_h + q_{1h}\overline{Y}^2_h\tau^2_h\left\{\frac{\delta_h}{2}\left(1 + \frac{\delta_h}{2}\right) - n^2_h g_h(1 + g_h) + \delta_h n_h g_h\right\} + 2q_{2h} + 4q_{3h}\overline{Y}_h\tau_h\left\{\frac{\delta_h}{2} + n_h g_h\right\}\right]$$

The optimum weights of ω_{1h} and ω_{2h} are given as

$$\omega_{1h(opt)} = \frac{2B_hC_h - D_hE_h}{E_h^2 - 4A_hB_h}$$

$$\omega_{2h(opt)} = \frac{2A_hD_h - C_hE_h}{E_h^2 - 4A_hB_h}$$

Minimum MSE is obtained by placing the optimum weights of ω_{1h} and ω_{2h} in equation (23)

$$MSE_{min}\,(T_{st}) \cong \sum_{h=1}^{L} W_h^2 \left[\bar{Y}_h - \frac{\left(C_hD_hE_h - B_hC_h^2 - A_hD_h^2\right)}{\left(E_h^2 - 4A_hB_h\right)} \right] \quad ...(24)$$

RESULTS AND DISCUSSION

A simulation study is conducted in this section to assess the empirical performance of proposed estimators as compared to other estimators. Two populations are generated through multivariate normal distribution under different parameters with the help of R language programme. Population I is generated for equal strata and population II is generated for unequal strata. Data statistics of population I and population II are shown in Tables 4 and 5. Equation (25) presents an expression to compute the percent relative efficiencies (PREs) of all the estimators with respect to the Hansen and Hurwitz (1946).

$$PRE = \frac{MSE\left(\bar{y}^*_{st(HH)}\right)}{MSE(\circledcirc)} \times 100 \quad ...(25)$$

where $\circledcirc = \bar{y}^*_{st(HH)}, \bar{y}_{st(KU)}, \bar{y}_{st(AHi)}, \bar{y}_{st(ZS)}, T^{(1)}_{sti}, T^{(2)}_{sti}, T^{(3)}_{sti}, T^{(4)}_{sti}, T^{(5)}_{sti}, T^{(6)}_{sti}, T^{(7)}_{sti}, T^{(8)}_{sti}$

Ten percent (10%) non-response is considered to calculate the PREs and are presented in Tables 6 and 7. Important findings from Tables 6 and 7 are summarized below.

- Kumar *et al*. (2015) and third estimator of Azeem and Hanif (2017) are equally efficient as in each case PRE of $\bar{y}_{st(KU)}$ = PRE of $\bar{y}_{st(AH3)}$.
- $\bar{y}_{st(ZS)}$ perform better than $\bar{y}^*_{st(HH)}, \bar{y}_{st(KU)}$ and $\bar{y}_{st(AHi)}$. Moreover, $\bar{y}_{st(ZS)}$ provides maximum PRE at $\gamma_h = 0$ in each case.
- It is worth pointing out that all the proposed series of estimators $T^{(1)}_{sti}, T^{(2)}_{sti}, T^{(3)}_{sti}, T^{(4)}_{sti}, T^{(5)}_{sti}, T^{(6)}_{sti}, T^{(7)}_{sti}, T^{(8)}_{sti}$ are more competent than $\bar{y}_{st(ZS)}$..

Therefore, the above findings confirmed that the proposed series of estimators out-perform than all competing estimators under study.

CONCLUSION

This paper emphasizes on the utilisation of non-conventional auxiliary information as well as conventional information for enhanced estimation of population mean in stratified random sampling under the combined effect of non-response and measurement error. Different series of ratio and product type estimators have been proposed in the present study. These series can generate a number of estimators by choosing different known auxiliary information (conventional and non-conventional). Expressions of bias and MSE of the proposed estimators are derived up to first degree of approximation. Using R language programme, a simulation study is conducted by generating two populations from multivariate normal distribution. First population is generated with equal strata and second population with unequal strata. Empirical performance of all the estimators under study is assessed in terms of PREs with respect to Hansen and Hurwitz (1946). Suggested estimators outperform than all other competing estimators. Therefore, in case of non-response and measurement errors under stratified random sampling it is suggested to apply the proposed estimators.

The possible extensions of this work are to estimate the: 1) finite population mean under other sampling designs like stratified double sampling, rank set sampling, etc. 2) other unknown finite population parameters including median, variance and proportions etc.

REFERENCES

Azeem M. & Hanif M. (2017). Joint influence of measurement error and non-response on estimation of population mean. *Communications in Statistics-Theory and Methods* **46**(4): 1679–1693.
DOI: https://doi.org/10.1080/03610926.2015.1026992

Cochran W. (1968). Errors of measurement in statistics. *Technometrics* **10**(4): 637–666.
DOI: https://doi.org/10.2307/1267450

Cochran W. (1977). *Sampling Techniques*, 3rd edition. John Wiley & Sons, Inc., New York, USA.

Hansen M.H. & Hurwitz W.N. (1946). The problem of non-response in sample surveys. *Journal of American Statistical Association* **41**: 517–529.
DOI: https://doi.org/10.1080/01621459.1946.10501894

Irfan M., Javed M. & Lin Z. (2018). Optimized estimation for population mean using conventional and non-conventional measures under the joint influence of measurement error and non-response. *Journal of Statistical Computation and Simulation* **88**(12): 2385–2403.
DOI: https://doi.org/10.1080/00949655.2018.1464571

Kumar S. (2015). Efficient use of auxiliary information in estimating the population ratio, product and mean in the presence of non-response. *Journal of Advanced Computing* **4**(2): 59–67.
DOI: https://doi.org/10.7726/jac.2015.1005

Kumar S. (2016). Improved estimation of population mean in presence of nonresponse and measurement error. *Journal of Statistical Theory and Practice* **10**(4): 707–720.
DOI: https://doi.org/10.1080/15598608.2016.1216488

Kumar S., Bhougal S., Nataraja N. S. & Viswanathaiah M. (2015). Estimation of population mean in the presence of non-response and measurement error. *Revista Colombiana de Estadistica* **38**(1): 145–161.
DOI: https://doi.org/10.15446/rce.v38n1.48807

Kumar S. & Chatterjee K. (2014). A simulation study for unit non-response under double sampling. *Journal of Statistics Applications and Probability Letters* **2**(1): 37–50.

Rao P. (1986). Ratio estimation with sub sampling the non-respondents. *Survey Methodology* **12**: 217–230.

Shalabh (1997). Ratio method of estimation in the presence of measurement errors. *Journal of the Indian Society of Agricultural Statistics* **50**: 150–155.

Sharma P. & Singh R. (2013). A generalized class of estimators for finite population variance in presence of measurement errors. *Journal of Modern Applied Statistical Methods* **12**: 231–241.
DOI: https://doi.org/10.22237/jmasm/1383279120

Shukla D., Pathak S. & Thakue N.S. (2012). Class(es) of factor-type estimator(s) in presence of measurement error. *Journal of Modern Applied Statistical Methods* **11**: 336–347.
DOI: https://doi.org/10.22237/jmasm/1351742700

Singh H.P. & Karpe N. (2009). On the estimation of ratio and product of two population means using supplementary information in presence of measurement errors. *Statistica* **69**: 27–47.
DOI: https://doi.org/10.6092/issn.1973-2201/3546

Singh H. P. & Karpe N. (2010). Estimation of mean, ratio and product using auxiliary information in the presence of measurement errors in sample surveys. *Journal of Statistical Theory and Practice* **4**: 111–136.
DOI: https://doi.org/10.1080/15598608.2010.10411976

Singh H.P. & Kumar S. (2008). A regression approach to the estimation of the finite population mean in the presence of non-response. *Australian and New Zealand Journal of Statistics* **50**: 395–408.
DOI: https://doi.org/10.1111/j.1467-842X.2008.00525.x

Singh H.P., Kumar S. & Kozak M. (2010). Improved estimation of finite population mean using sub-sampling to deal with the non-response in two-phase sampling scheme. *Communications in Statistics-Theory and Methods* **39**(5): 791–802.
DOI: https://doi.org/10.1080/03610920902796056

Zahid E. & Shabbir J. (2018). Estimation of population mean in the presence of measurement error and nonresponse under stratified random sampling. *PLOS ONE* **13**(2): e0191572.
DOI: https://doi.org/10.1371/journal.pone.0191572

Appendix

Table 2: Generation of series with proposed estimator T_{st}

Values of $(\omega_{1h}, \omega_{2h}, g_h, \delta_h)$				Series of estimators	
ω_{1h}	ω_{2h}	g_h	δ_h	Estimator	Name
ω_{1h}	ω_{2h}	1	0	$T_{sti}^{(1)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{\bar{Z}_h}{\bar{z}_h}\right) + \omega_{2h}\bar{y}_h^*\right]$	Ratio-type
ω_{1h}	ω_{2h}	-1	0	$T_{sti}^{(2)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{\bar{z}_h}{\bar{Z}_h}\right) + \omega_{2h}\bar{y}_h^*\right]$	Product-type
ω_{1h}	ω_{2h}	0	1	$T_{sti}^{(3)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* + \omega_{2h}\bar{y}_h^* \exp\left(\frac{\bar{Z}_h - \bar{z}_h}{\bar{Z}_h + \bar{z}_h}\right)\right]$	Ratio-type exponential
ω_{1h}	ω_{2h}	0	-1	$T_{sti}^{(4)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* + \omega_{2h}\bar{y}_h^* \exp\left(\frac{\bar{z}_h - \bar{Z}_h}{\bar{Z}_h + \bar{z}_h}\right)\right]$	Product-type exponential
ω_{1h}	ω_{2h}	1	1	$T_{sti}^{(5)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{\bar{Z}_h}{\bar{z}_h}\right) + \omega_{2h}\bar{y}_h^* \exp\left(\frac{\bar{Z}_h - \bar{z}_h}{\bar{Z}_h + \bar{z}_h}\right)\right]$	Ratio-ratio-type exponential
ω_{1h}	ω_{2h}	-1	-1	$T_{sti}^{(6)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{\bar{z}_h}{\bar{Z}_h}\right) + \omega_{2h}\bar{y}_h^* \exp\left(\frac{\bar{z}_h - \bar{Z}_h}{\bar{Z}_h + \bar{z}_h}\right)\right]$	Product-product-type exponential
ω_{1h}	ω_{2h}	1	-1	$T_{sti}^{(7)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{\bar{Z}_h}{\bar{z}_h}\right) + \omega_{2h}\bar{y}_h^* \exp\left(\frac{\bar{z}_h - \bar{Z}_h}{\bar{Z}_h + \bar{z}_h}\right)\right]$	Ratio-product-type exponential
ω_{1h}	ω_{2h}	-1	1	$T_{sti}^{(8)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{\bar{z}_h}{\bar{Z}_h}\right) + \omega_{2h}\bar{y}_h^* \exp\left(\frac{\bar{Z}_h - \bar{z}_h}{\bar{Z}_h + \bar{z}_h}\right)\right]$	Product-ratio-type exponential

Table 3: Some members of $T_{sti}^{(1)}$ and $T_{sti}^{(3)}$

			Ratio-type estimators	Ratio-type exponential
a_h	b_h	i	$T_{sti}^{(1)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{a_h\bar{X}_h + b_h}{a_h\acute{\bar{x}}_h^* + b_h}\right) + \omega_{2h}\bar{y}_h^*\right]$	$T_{sti}^{(3)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* + \omega_{2h}\bar{y}_h^* \exp\left(\frac{a_h(\bar{X}_h - \acute{\bar{x}}_h^*)}{a_h(\bar{X}_h + \acute{\bar{x}}_h^*) + 2b_h}\right)\right]$
1	C_{Xh}	1	$T_{st1}^{(1)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{\bar{X}_h + C_{Xh}}{\acute{\bar{x}}_h^* + C_{Xh}}\right) + \omega_{2h}\bar{y}_h^*\right]$	$T_{st1}^{(3)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* + \omega_{2h}\bar{y}_h^* \exp\left(\frac{\bar{X}_h - \acute{\bar{x}}_h^*}{\bar{X}_h + \acute{\bar{x}}_h^* + 2C_{Xh}}\right)\right]$
$\beta_{2h(X)}$	C_{Xh}	2	$T_{st2}^{(1)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{\beta_{2h}(X)\bar{X}_h + C_{Xh}}{\beta_{2h}(X)\acute{\bar{x}}_h^* + C_{Xh}}\right) + \omega_{2h}\bar{y}_h^*\right]$	$T_{st2}^{(3)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* + \omega_{2h}\bar{y}_h^* \exp\left(\frac{\beta_{2h(X)}(\bar{X}_h - \acute{\bar{x}}_h^*)}{\beta_{2h(X)}(\bar{X}_h + \acute{\bar{x}}_h^*) + 2C_{Xh}}\right)\right]$
1	ρ_{YXh}	3	$T_{st3}^{(1)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{\bar{X}_h + \rho_{YXh}}{\acute{\bar{x}}_h^* + \rho_{YXh}}\right) + \omega_{2h}\bar{y}_h^*\right]$	$T_{st3}^{(3)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* + \omega_{2h}\bar{y}_h^* \exp\left(\frac{\bar{X}_h - \acute{\bar{x}}_h^*}{\bar{X}_h + \acute{\bar{x}}_h^* + 2\rho_{YXh}}\right)\right]$
C_{Xh}	ρ_{YXh}	4	$T_{st4}^{(1)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{C_{Xh}\bar{X}_h + \rho_{YXh}}{C_{Xh}\acute{\bar{x}}_h^* + \rho_{YXh}}\right) + \omega_{2h}\bar{y}_h^*\right]$	$T_{st4}^{(3)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* + \omega_{2h}\bar{y}_h^* \exp\left(\frac{C_{Xh}(\bar{X}_h - \acute{\bar{x}}_h^*)}{C_{Xh}(\bar{X}_h + \acute{\bar{x}}_h^*) + 2\rho_{YXh}}\right)\right]$
S_{Xh}	$\beta_{1h(X)}$	5	$T_{st5}^{(1)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{S_{Xh}\bar{X}_h + \beta_{1h}(X)}{S_{Xh}\acute{\bar{x}}_h^* + \beta_{1h}(X)}\right) + \omega_{2h}\bar{y}_h^*\right]$	$T_{st5}^{(3)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* + \omega_{2h}\bar{y}_h^* \exp\left(\frac{S_{Xh}(\bar{X}_h - \acute{\bar{x}}_h^*)}{S_{Xh}(\bar{X}_h + \acute{\bar{x}}_h^*) + 2\beta_{1h(X)}}\right)\right]$
HL_h	C_{Xh}	6	$T_{st6}^{(1)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{HL_h\bar{X}_h + C_{Xh}}{HL_h\acute{\bar{x}}_h^* + C_{Xh}}\right) + \omega_{2h}\bar{y}_h^*\right]$	$T_{st6}^{(3)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* + \omega_{2h}\bar{y}_h^* \exp\left(\frac{HL_h(\bar{X}_h - \acute{\bar{x}}_h^*)}{HL_h(\bar{X}_h + \acute{\bar{x}}_h^*) + 2C_{Xh}}\right)\right]$
C_{Xh}	TM_h	7	$T_{st7}^{(1)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{C_{Xh}\bar{X}_h + TM_h}{C_{Xh}\acute{\bar{x}}_h^* + TM_h}\right) + \omega_{2h}\bar{y}_h^*\right]$	$T_{st7}^{(3)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* + \omega_{2h}\bar{y}_h^* \exp\left(\frac{C_{Xh}(\bar{X}_h - \acute{\bar{x}}_h^*)}{C_{Xh}(\bar{X}_h + \acute{\bar{x}}_h^*) + 2TM_h}\right)\right]$
MR_h	ρ_{YXh}	8	$T_{st8}^{(1)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{MR_h\bar{X}_h + \rho_{YXh}}{MR_h\acute{\bar{x}}_h^* + \rho_{YXh}}\right) + \omega_{2h}\bar{y}_h^*\right]$	$T_{st8}^{(3)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* + \omega_{2h}\bar{y}_h^* \exp\left(\frac{MR_h(\bar{X}_h - \acute{\bar{x}}_h^*)}{MR_h(\bar{X}_h + \acute{\bar{x}}_h^*) + 2\rho_{YXh}}\right)\right]$
ρ_{YXh}	HL_h	9	$T_{st9}^{(1)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{\rho_{YXh}\bar{X}_h + HL_h}{\rho_{YXh}\acute{\bar{x}}_h^* + HL_h}\right) + \omega_{2h}\bar{y}_h^*\right]$	$T_{st9}^{(1)} = \sum_{h=1}^{L} W_h \left[\omega_{1h}\bar{y}_h^* \left(\frac{\rho_{YXh}\bar{X}_h + HL_h}{\rho_{YXh}\acute{\bar{x}}_h^* + HL_h}\right) + \omega_{2h}\bar{y}_h^*\right]$
			h^{th}	um
		10		
			1 2	3 4

Table 4: Data statistics population I

	h^{th} stratum											
h		1			2			3			4	
X_h		$rnorm(1000, 5, 10)$,			$rnorm(1000, 4, 8)$,			$rnorm(1000, 4, 9)$,			$rnorm(1000, 3, 7)$,	
Y_h		$X_1 + rnorm(1000, 0,1)$			$X_2 + rnorm(1000, 0,1)$			$X_3 + rnorm(1000, 0,1)$			$X_4 + rnorm(1000, 0,1)$	
y_h		$Y_1 + rnorm(1000, 1,3)$			$Y_2 + rnorm(1000, 1,3)$			$Y_3 + rnorm(1000, 1,3)$			$Y_4 + rnorm(1000, 1,3)$	
x_h		$X_1 + rnorm(1000, 1,3)$			$X_2 + rnorm(1000, 1,3)$			$X_3 + rnorm(1000, 1,3)$			$X_4 + rnorm(1000, 1,3)$	
$\overline{Y}_h$		4.887			3.801			4.003			3.093	
$\overline{X}_h$		4.911			3.757			4.044			3.128	
S_{Yh}^2		97.315			66.002			81.220			51.932	
S_{Xh}^2		97.036			64.658			80.508			50.357	
S_{Uh}^2		9.092			9.018			9.439			9.615	
S_{Vh}^2		9.590			9.233			9.735			9.124	
ρ_{YXh}		0.995			0.993			0.994			0.991	
$\beta_{1h(X)}$		-0.06			-0.11			0.09			0.08	
$\beta_{2h(X)}$		-0.12			-0.03			0.03			-0.02	
C_{Xh}		2.006			2.140			2.219			2.269	
HL_h		5.034			3.668			4.106			3.098	
QD_h		6.727			5.119			5.878			4.930	
MR_h		4.262			4.292			2.885			3.045	
TM_h		4.965			3.815			3.986			3.093	
Non-response	10%	15%	20%	10%	15%	20%	10%	15%	20%	10%	15%	20%
$S_{Y(2)h}^2$	97.444	102.288	99.046	68.973	64.563	68.963	86.179	80.096	77.793	48.614	51.898	49.204
$S_{X(2)h}^2$	98.454	103.120	99.098	67.857	62.385	68.860	82.651	77.998	77.284	46.823	51.404	46.827
$S_{U(2)h}^2$	10.877	8.668	8.725	7.707	10.469	8.587	10.150	7.973	9.853	10.501	10.015	11.136
$S_{V(2)h}^2$	11.388	9.182	9.519	9.341	8.745	9.865	8.966	12.068	9.159	11.918	10.736	8.997
$\rho_{YX(2)h}$	0.995	0.996	0.995	0.993	0.994	0.994	0.994	0.994	0.994	0.987	0.991	0.990

Table 5: Data statistics population II

h	1			2			3			4		
X_h	$rnorm(1000, 5, 10)$,			$rnorm(1200, 4, 8)$,			$rnorm(1300, 4, 9)$,			$rnorm(1500, 3, 7)$,		
Y_h	$X_1 + rnorm(1000, 0,1)$			$X_2 + rnorm(1200, 0,1)$			$X_3 + rnorm(1300, 0,1)$			$X_4 + rnorm(1500, 0,1)$		
y_h	$Y_1 + rnorm(1000, 1,3)$			$Y_2 + rnorm(1200, 1,3)$			$Y_3 + rnorm(1300, 1, 3)$			$Y_4 + rnorm(1500, 1, 3)$		
x_h	$X_1 + rnorm(1000, 1,3)$			$X_2 + rnorm(1200, 1,3)$			$X_3 + rnorm(1300, 1,3)$			$X_4 + rnorm(1500, 1,3)$		
n_h	200			240			260			300		
$\overline{Y}_h$	5.157			3.649			3.563			2.765		
$\overline{X}_h$	5.148			3.686			3.554			2.756		
S^2_{Yh}	99.459			60.788			88.185			49.353		
S^2_{Xh}	98.151			59.113			86.809			48.277		
S^2_{Uh}	9.152			8.882			8.829			9.426		
S^2_{Vh}	9.666			8.821			9.003			8.775		
ρ_{YXh}	0.995			0.992			0.994			0.990		
$\beta_{1h(X)}$	-0.05			0.10			-0.05			0.03		
$\beta_{2h(X)}$	0.01			0.05			0.07			0.07		
C_{Xh}	1.924			2.086			2.622			2.521		
HL_h	5.125			3.671			3.468			2.605		
QD_h	6.765			5.093			6.172			4.714		
MR_h	4.781			4.493			2.572			2.685		
TM_h	5.196			3.630			3.569			2.734		
Non-response	10%	15%	20%	10%	15%	20%	10%	15%	20%	10%	15%	20%
$S^2_{Y(2)h}$	113.600	112.505	107.682	59.723	65.300	67.144	82.390	76.975	78.004	51.278	50.815	51.802
$S^2_{X(2)h}$	112.810	111.589	105.596	60.529	65.222	64.166	81.073	78.085	78.551	50.360	49.828	50.710
$S^2_{U(2)h}$	9.279	8.273	8.405	8.980	9.110	9.794	8.216	8.509	7.170	9.561	9.506	11.108
$S^2_{V(2)h}$	10.708	10.735	10.324	10.005	10.349	10.193	9.362	8.300	8.704	9.307	8.843	9.860
$\rho_{YX(2)h}$	0.996	0.995	0.994	0.993	0.992	0.992	0.994	0.993	0.994	0.991	0.990	0.991

Table 6: Percent relative efficiencies of the competing estimators w.r.t. $\bar{y}^*_{st(HH)}$ for population I (10 % non-response)

				Estimator										
j_h	$\bar{y}^*_{st(HH)}$	$\bar{y}_{st(KU)}$	γ_h	$\bar{y}_{st(ZS)}$	i	$\bar{y}_{st(AHi)}$	$T^{(1)}_{sti}$	$T^{(2)}_{sti}$	$T^{(3)}_{sti}$	$T^{(4)}_{sti}$	$T^{(5)}_{sti}$	$T^{(6)}_{sti}$	$T^{(7)}_{sti}$	$T^{(8)}_{sti}$
2	100	446.686	-1	449.288	1	168.961	490.400	539.244	539.489	539.326	490.294	539.244	490.505	539.245
			0	449.459	2	445.407	558.076	539.244	539.223	539.148	558.125	539.245	558.027	539.244
			1	449.292	3	446.686	483.122	539.244	539.542	539.343	483.005	539.244	483.238	539.245
					4		478.999	539.244	539.576	539.355	478.877	539.244	479.120	539.245
					5		474.848	539.244	539.613	539.367	474.722	539.244	474.974	539.245
					6		479.401	539.244	539.572	539.354	479.279	539.244	479.522	539.245
					7		489.625	539.244	539.495	539.328	489.519	539.244	489.732	539.245
					8		477.384	539.244	539.590	539.359	477.260	539.244	477.507	539.245
					9		500.151	539.244	539.427	539.306	500.063	539.244	500.239	539.245
					10		497.868	539.244	539.441	539.310	497.775	539.244	497.960	539.245
4	100	442.855	-1	446.051	1	167.925	498.617	567.833	568.198	567.955	498.475	567.833	498.759	567.834
			0	446.259	2	441.477	596.563	567.833	567.782	567.689	596.640	567.833	596.486	567.833
			1	446.057	3	442.855	488.933	567.833	568.277	567.981	488.778	567.833	489.086	567.834
					4		483.518	567.833	568.328	567.998	483.358	567.833	483.677	567.834
					5		478.117	567.833	568.383	568.016	477.953	567.833	478.279	567.834
					6		484.023	567.833	568.322	567.996	483.863	567.833	484.181	567.834
					7		497.612	567.833	568.206	567.957	497.469	567.833	497.755	567.834
					8		481.411	567.833	568.349	568.005	481.249	567.833	481.571	567.834
					9		511.890	567.833	568.106	567.923	511.769	567.833	512.010	567.834
					10		508.715	567.833	568.127	567.931	508.589	567.833	508.841	567.834
8	100	438.473	-1	442.859	1	166.707	524.762	664.117	664.969	664.400	524.516	664.116	525.006	664.119
			0	443.141	2	436.883	736.893	664.117	663.864	664.097	737.109	664.117	736.679	664.116
			1	442.870	3	438.473	508.319	664.117	665.154	664.462	508.063	664.116	508.575	664.119
					4		499.408	664.117	665.271	664.501	499.148	664.116	499.666	664.119
					5		490.712	664.117	665.400	664.544	490.452	664.116	490.971	664.119
					6		500.184	664.117	665.259	664.497	499.924	664.116	500.442	664.119
					7		523.106	664.117	664.987	664.406	522.859	664.116	523.351	664.119
					8		495.997	664.117	665.319	664.517	495.737	664.116	496.257	664.119
					9		548.525	664.117	664.752	664.328	548.304	664.116	548.744	664.118
					10		542.573	664.117	664.803	664.345	542.346	664.116	542.800	664.118

Table 7: Percent relative efficiencies of the competing estimators w.r.t. $\bar{y}^*_{st(HH)}$ for population II (10 % non-response)

					Estimator									
j_h	$\bar{y}^*_{st(HH)}$	$\bar{y}_{st(KU)}$	γ_h	$\bar{y}_{st(ZS)}$	i	$\bar{y}_{st(AHi)}$	$T^{(1)}_{sti}$	$T^{(2)}_{sti}$	$T^{(3)}_{sti}$	$T^{(4)}_{sti}$	$T^{(5)}_{sti}$	$T^{(6)}_{sti}$	$T^{(7)}_{sti}$	$T^{(8)}_{sti}$
	100	449.713	-1	452.051	1	169.061	492.206	533.996	534.175	534.056	492.125	533.996	492.286	533.997
			0	452.207	2	448.073	528.951	533.996	534.128	534.067	528.941	533.996	528.962	533.996
			1	452.054	3	449.713	484.637	533.996	534.217	534.070	484.547	533.996	484.728	533.997
					4		480.487	533.996	534.243	534.079	480.392	533.996	480.582	533.997
2					5		476.737	533.996	534.271	534.088	476.637	533.996	476.836	533.997
					6		481.964	533.996	534.235	534.075	481.869	533.996	482.058	533.997
					7		489.614	533.996	534.184	534.058	489.531	533.996	489.697	533.997
					8		479.412	533.996	534.252	534.082	479.315	533.996	479.508	533.997
					9		499.248	533.996	534.133	534.043	499.180	533.996	499.316	533.997
					10		497.682	533.996	534.141	534.044	497.612	533.996	497.753	533.997
	100	449.635	-1	452.499	1	168.209	505.438	566.401	566.676	566.492	505.326	566.401	505.550	566.402
			0	452.692	2	448.030	558.948	566.401	566.468	566.400	558.933	566.401	558.963	566.401
			1	452.504	3	449.635	495.186	566.401	566.740	566.514	495.061	566.401	495.310	566.402
					4		489.626	566.401	566.780	566.527	489.497	566.401	489.755	566.402
4					5		484.602	566.401	566.822	566.541	484.469	566.401	484.735	566.402
					6		491.504	566.401	566.769	566.524	491.375	566.401	491.632	566.402
					7		502.174	566.401	566.688	566.497	502.059	566.401	502.288	566.402
					8		488.131	566.401	566.793	566.532	488.000	566.401	488.262	566.402
					9		515.510	566.401	566.613	566.472	515.414	566.401	515.606	566.402
					10		513.338	566.401	566.623	566.474	513.238	566.401	513.437	566.402
	100	449.570	-1	453.488	1	167.203	538.909	665.877	666.543	666.098	538.704	665.877	539.113	665.879
			0	453.753	2	447.979	649.250	665.877	665.970	665.847	649.217	665.877	649.283	665.878
			1	453.497	3	449.570	521.153	665.877	666.694	666.149	520.936	665.877	521.369	665.879
					4		511.803	665.877	666.789	666.181	511.582	665.877	512.023	665.879
8					5		503.421	665.877	666.888	666.214	503.199	665.877	503.643	665.879
					6		514.693	665.877	666.766	666.173	514.472	665.877	514.913	665.879
					7		533.913	665.877	666.566	666.107	533.707	665.877	534.119	665.879
					8		509.171	665.877	666.823	666.192	508.949	665.877	509.392	665.879
					9		558.191	665.877	666.383	666.047	558.007	665.877	558.373	665.878
					10		554.145	665.877	666.411	666.055	553.958	665.877	554.332	665.879